# A PERSONALIZED SYSTEM OF INSTRUCTION FOR GAS DYNAMICS

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# THESIS

A PERSONALIZED SYSTEM OF INSTRUCTION
FOR GAS DYNAMICS

by

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December 1972

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# A Personalized System of Instruction

# for Gas Dynamics

by

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## ABSTRACT

A personalized system of instruction combining the principle of auto-tutorial instruction with modified self-pacing was applied to a course in the fundamentals of gas dynamics taught to thirteen students in the Department of Aeronautics at the Naval Postgraduate School, Monterey, during the Spring quarter of 1972. The course results are summarized and the instructional materials developed for the course are included as appendices. While some degradation was apparent due to modifications made to the method, the results tended to affirm the substantial value of this instructional method and its superiority to the conventional lecture method.

TABLE OF CONTENTS

## ACKNOWLEDGEMENTS

The author wishes to express his appreciation to Associate Professor Robert D. Zucker for his patience and guidance as advisor to the author in the development of this thesis.

## I. THE EDUCATION PROBLEM

Faced with the spectre of the "post-industrial age" [Ref. 1] with its exponential rise in population and complexity of life, the engineering profession is being tasked to solve problems of vastly increased scope with reduced assets. Rising costs, increasingly critical public scrutiny, and a disturbing downward trend in the number of candidates for engineering training are indicative of an impending crisis in technical education. Some of the more astute observers, including Vice Admiral Hyman Rickover, have been forecasting such problems for more than a decade [Ref. 2]. Recognizing these trends, engineering educators are urgently exploring new methods of instruction which will not only provide more depth to the engineering student's education, but will also provide for more efficient time utilization by both students and teachers.

R. Buckminster Fuller maintains that it will not suffice to merely expand our present educational systems, but rather that a "transformation" of great magnitude is essential [Ref. 3]. The early stages of this transformation are already in progress and have provided some of the impetus and direction to this project.

Innovations in engineering education almost invariably represent a departure from the venerable and currently prevalent mode of teaching known as "lecturing," wherein the

instructor devotes a major portion of his contact hours with the students to the role described by Professor Flammer of Utah State as that of a "dispenser of information" [Ref. 4]. He sees a dire need for the role of the professor to change — not diminishing as some observers fear with the advent of various other media for the dispensing of information, but rather assuming vastly greater importance as the teacher becomes more of a <u>diagnostician</u> and <u>prescriber of learning experiences</u> for each individual student [Ref. 4]. As Dr. Billy Koen has suggested, "... it is not unreasonable to expect that engineering educators ought to be able to 'engineer' their courses." [Ref. 5]

Learning may be thought of as resulting from the auspicious combination of several interactive factors which can either promote or hamper successful learning. The three principal influences are <u>motivation</u>, <u>operant span</u> and <u>stimulus control</u>. Each of these is in turn influenced to a greater or lesser degree by several other educational factors. To draw an analogy from the realm of control theory, the problem in education is to introduce methods which generate feedback in order to modify the influence of each of these independent factors, so as to optimize learning. One basic premise of the concept of 'personalized instruction' is that this more effectively provides the required feedback.

The influencing factors are defined as follows:

<u>Motivation</u> - The force which drives a student to seek to learn.

Stimulus Control - The effect exerted by the learning surrounds on the individual.

Operant Span - The maximum scope (in size or complexity) of a single block of material which may be assimilated by a given student at a time.

Structure - The concept and method of presentation of the course material.

Facilities - The physical surrounds (including graphic aids and other media).

Pacing - Rate of delivery.

Intellectual Ability - The student's inherent talent.

Level of Development - The student's specific level of achievement or skill in a given subject area.

Quotidian Variability - A psychometric term used to identify daily variations in the student, not intrinsic to the educational method, such as illness, fatigue, or behavior arising from idiosyncratic response to some unique aspect or feature of the educational environs. [Ref. 6]

The interaction of these factors forms the basis of a substantial portion of current educational research. Detailed investigation of these interactions represents subject matter well beyond the scope of this paper. Structure and pacing are the most obvious areas of concern in a system of personalized instruction, though each factor must be considered.

In the development of better methods, two unique aspects of engineering education must be considered:

a. The professional skills and concepts involve principally the cognitive domain.

b. A hierarchy of skills exists, which causes mastery of most areas to be dependent on a thorough grounding in preceding material.

Failure to thoroughly master the required hierarchy of skills (a common tendency among many students) can lead to the development of a condition referred to as "cumulative ignorance," which probably accounts for the majority of the failures and "drop outs" in the engineering educational process. If no other gain be realized, the possibility of precluding some cumulative ignorance justifies the efforts expended in searching for better methods. However there is mounting evidence that methods have been devised and are proven which not only help solve the problem of cumulative ignorance, but also provide far more optimal interaction of the various parameters affecting the learning process.

## II. THE CONVENTIONAL LECTURE METHOD

The standard with which all other educational methods are usually compared (or contrasted) is the "conventional lecture method." The term as used herein connotes a formal discourse wherein the instructor ("lecturer") presents the course material through the media of the spoken word, using no materials or training aids other than a chalk board. The lecture method normally exhibits the following characteristics:

a. The lecture is time-constrained by the standard scheduled period.

b. The student is in a passive role and is expected to transcribe by notes, or otherwise record, whatever information he deems essential.

c. The pace is essentially fixed by the lecturer based on the scheduled contact hours/course coverage.

d. There is virtually no "real-time" feedback. Due to the time constraints, questions or individual difficulties are usually deferred. This leads to the not unusual situation of the student missing vital succeeding concepts while momentarily pondering a puzzling point.

e. The clarity of presentation is strongly dependent on the expository ability of the lecturer.

The conventional lecture method is seldom utilized exactly as described above. It is normally modified by one (or more) of many variations in structure or facilities ranging from simple devices, such as the use of colored chalk, to the complex, such as closed circuit television with printed, pre-distributed notes. Where class size and time permit, most lecturers seek feedback by encouraging questions from the class at any time. While these variations mitigate somewhat the shortcomings of the lecture, the basic weaknesses remain.

The more capable students are held back and the students suffering from any substantial amount of "cumulative ignorance" find their burden of cumulative ignorance is growing.

The lecture method has three advantages which have acted to foster its acceptance for hundreds, if not thousands of years. On closer examination these 'so-called' advantages prove to contribute to administrative expediency - but often at the expense of learning. There is truth in the statement that students often learn the material of a course in <u>spite</u> of the teaching methods rather than <u>because</u> of them. Firstly, the conventional lecture provides for the most rapid and inexpensive output of information by the lecturer with the minimum amount of equipment or materials. This says nothing about the amount of information actually received by the student. Secondly, the elegance of presentation and the influence of the lecturer on his class can serve to "sell" the concepts involved to the class. Thirdly, this method is accepted. This is a very real consideration when one must deal with the problem of institutional inertia.

## III. ALTERNATIVES TO THE CONVENTIONAL LECTURE METHOD

Many of the variations or innovations associated with non-lecture methods, such as written course objectives, instructor tutoring, peer tutoring, visual aids and printed notes or outlines, can be effectively used to enhance the lecture. However, while improving the lecture method, these innovations do not directly contribute to the basic need of each student to explore the topic at his own best speed. This can only be achieved by abandoning the lecture as the primary information vehicle.

Currently in vogue among engineering educators are several varieties of non-lecture methods, nearly all based on the reinforcement theories first expounded by Sidney Pressey and B. F. Skinner [Ref. 7]. Their research resulted first in the development of "programmed instruction". The Programmed Text and the Teaching Machine are direct applications of this reinforcement theory and are now widely used (with proven effectiveness) for the instruction of material involving verbal or skill knowledge. These methods suffer from relatively inflexible structure and the tendency to design the instructor out of the picture. Also, neither method has proven quite so adaptable to complex concepts involving cognitive strategy, such as engineering problem solving or design. For this reason, engineering educators have been among the last to experiment with new teaching techniques. However, these shortcomings have recently

prompted the development of several closely related methods known variously as "Auto-Tutorial Instruction" [Ref. 8], "Self-Paced Individually Prescribed Instruction" [Ref. 9], "Personalized (or Proctorial) System of Instruction"[Ref. 10, 11] or some similar designation, all of which are more or less <u>self-paced</u> and vary primarily in the method by which reinforcement is achieved and progress is evaluated. However, in all cases the <u>teacher</u>, in the role of diagnostician, tutor, counselor and designer of learning experiences [Ref. 12] is the <u>key</u> to success of these systems.

## IV. PERSONALIZED INSTRUCTION

Professor F. H. Keller is generally regarded as the principal architect of this method which he calls the "Proctorial (or personalized) System of Instruction" (PSI), though many of the features were concurrently and independently developed by Professor S. H. Postlewaite at Purdue University in Indiana. The basic features of this method are succinctly described by Professor B. V. Koen of the University of Texas as follows: [Ref. 13]

1. The 'go-at-your-own-pace' feature, which permits a student to move through the course at a speed commensurate with his ability and other demands upon his time;
2. The unit-perfection requirement for advance, which lets the student go ahead to new material only after demonstrating mastery of that which preceded;
3. The use of lectures as vehicles of motivation, rather than sources of critical information;
4. The related stress upon the written work in teacher-student communication; and, finally:
5. The use of proctors, which permits repeated testing, immediate scoring, almost unavoidable tutoring, and a marked enhancement of the personal-social aspect of the educational process.

This summary by Dr. Koen clearly illustrates the particular desirability and applicability of this method to engineering education and accounts for the growing success of this method. The particularly agonizing problem of "cumulative ignorance" is effectively overcome by the unit-perfection requirement of this method.

The concept of PSI is predicated on the validity of '<u>reinforcement theory</u>.' The course is divided into an

appropriate number of units, depending on major subject areas and complexity of the material. Each student is provided with a Study Guide for the unit of instruction on which he is working which will contain:

a. Course objectives - defined in terms of measurable behavior.

b. Reading assignments (if a text is used).

c. Discussion of theoretical concepts and application to specific situations.

d. Example problems with a discussion of the general philosophy and specific techniques of solution.

e. Practice problems, with answers and guidance where necessary.

The student's initial exposure to new concepts can thus proceed at his own pace. A fundamental concept of PSI is that the instructor holds the key to success of the system in his role as "trouble-shooter." Ideally the Study Guides will provide a sufficiently clear exposition of the concepts so that the teacher will be freed to spend most of his time dealing with the individual problems which require <u>personal</u> attention.

The principal problems encountered by practitioners of this method were summarized recently by Dr. Gilmour Sherman of Georgetown University — who, as an associate of Professor Keller in the development of the PSI method, has received considerable feedback from various educators implementing PSI in one or another form. Dr. Sherman's list can be summarized as follows: [Ref. 14]

1. Professor works harder than ever.
2. Students work harder.
3. Substantial increase in logistic and administrative workload.

4. Staff (if paid) becomes expensive.
5. Class size for effective implementation is limited.
6. Production lead time for materials is burdensome.
7. Procrastination by students creates problems with the administration.
8. How to assign grades?

Of these, the last two problem areas seem the most pernicious. The advocates of PSI with experience have generally shown that the system can be engineered to minimize all of these problems with a few iterations, resulting in proven increases in effectiveness of instruction. For an in-depth current assessment of the Keller Method of PSI, the reader is referred to Ref. 15, in its entirety.

## V. APPLICATION OF PSI TO A GAS DYNAMICS COURSE AT NPS

An initial consideration in the development of PSI materials for the Gas Dynamics course offered at NPS was to insure compatibility with the quarter system. It was therefore determined that while retaining the self-paced, self-guided features of the course for each instructional unit, the pace would be controlled in a longterm sense by scheduling the check tests (to determine mastery of each unit) to be taken at regular intervals by all (13) students involved in the PSI course. This also permitted adequate lead time for the development of the Study Guides for each unit and allowed some feedback of information to modify later units to improve any notable weaknesses in format discovered in early units. The study guides are included as Appendix A.

Two major mid-course modifications were instituted. The first, a decision to discontinue the practice of repeating check tests (except in an extreme case) was made by the instructor when it was determined that time constraints may have prevented the class from finishing within the quarter. Secondly, in order to appease a number of students who found it discomforting to have no group sessions, the instructor instituted the practice of group review discussions in order to re-inforce the material. This provided tutoring on a group basis, for those who desired to attend the problem sessions.

## VI. EVALUATION OF RESULTS

The success of this project was intended initially to be measured by three principal means:

1. Comparison with a "control" group in the form of a class of twelve students taught by the instructor using the conventional lecture method during the same quarter.
2. A comprehensive student questionaire administered at the end of the course to the thirteen students involved in the "PSI" class.
3. A critique session concerning the course, to be moderated by the writer at the end of the quarter.

Due to the lack of statistical significance of any data involving groups of this size, grade comparisons are considered to have little meaning. However the instructor felt that there was a tendency toward better overall comprehension by the students in the PSI group as determined by major quiz and final examination averages.

The questionaire (Appendix B) which was given to each student in the PSI group revealed several key facts. Twelve of the thirteen students returned the questionaire and where appropriate the numerical average is tabulated in the right hand column beside the question. Noteworthy are the student's apparent high regard for the concept of defined objectives, study guides and check tests. Significant comments from the last questions are included in Appendix C.

The critique session tended to corroborate the comments in the questionaires. A noteworthy comment from this session is included in Appendix C.

## VII. CONCLUSIONS AND RECOMMENDATIONS

In-so-far as this course followed the model of PSI as conceived by Professor Keller, the results, based on the subjective evaluation of the students involved and the instructor, tend to substantiate the value of this approach. Given the bastardization of the long-term self-paced concept which resulted from controlling the administration of the check tests in the manner described, the hoped-for uniform high level of mastery was not achieved. However, several comments by students in the course indicated the 'depth' of knowledge achieved was generally greater than normal and most students admitted they worked harder than in previous courses. The controls applied to the pacing of the course negated a true evaluation of the problem of student procrastination; however, comments from two students who fared poorly in the course attributed this to a personal lack of interest in the subject area. It is also noteworthy that only four of the twelve students who returned questionaires sought tutorial aid from the instructor. This resulted in part from the students' dependence on the review discussion sessions to iron out problems and in at least one instance was the admitted result of a student's personal antipathy toward the course and instructor. This is a fairly obvious example of quotidian variability adversely affecting motivation, hence learning. The student in question is otherwise average but fared poorly in this course.

It is recommended that in the next offering of Gas-Dynamics the course be made truly self-paced with the use of a proctor drawn from one of the students who have taken the course under this project. Eleven of the twelve students who responded to the questionaire indicated their willingness to serve as a tutor. While it is not unreasonable to assume that any student volunteering to serve in this capacity would be substantially compensated by the challenge involved in this task, the time constraints and the inherent demands of the academic load certainly justify some more tangible reward. Many institutions are paying the graduate students utilized in this capacity. This being inappropriate at this institution, it is strongly recommended that some sort of academic credit be granted to the student undertaking this task as tutor or proctor.

With the materials and experience resulting from this first cycle, the foundation exists to truly exploit this proven method in succeeding offerings of the Gas Dynamics course at the Naval Post-graduate school. Moreover the flexibility and effectiveness of this method indicate the desirability of applying this method to other courses. Perhaps the most concise statement to date concerning the underlying strength of the Keller method was made by Professor A. J. Dressler in the foreword to a recent workshop on the Keller Method. [Ref. 15]

"The rapid spread of the Keller method is remarkable when one considers the handicaps it has had to overcome:

(1) It is a new idea that challenges a teaching technique (the lecture method) that is hundreds, if not thousands, of years old. New ideas are not usually received with delight.

(2) It is based on a controversial theory; the reinforcement theory associated with the name of B. F. Skinner.

(3) It involves more work for both the teacher and the student. More work is something that most people feel they do not need.

In spite of these negative features, the Keller method has spread at a remarkable rate for one simple reason - - it works!"

## APPENDIX A - STUDY GUIDES

The Study Guides written for this course were developed to be used in conjunction with the text *Gas Dynamics* by Professor James E. A. John [Ref. 16]. Additional reference material and assistance was gained from Refs. 17 - 20. The Course Objectives were developed using the methods in Refs. 21 and 22.

Some additional handouts were prepared by the instructor and distributed to assist the students with certain points. Some of this material will eventually be incorporated in the future revisions of the Study Guides.

## INTRODUCTION

It is assumed that before entering the world of Gas Dynamics you have had a reasonable background in Mathematics (through Calculus), together with an exposure to basic Fluid Dynamics and a course in elementary Thermodynamics.

Prior to undertaking new material it will prove fruitful to review some fundamental concepts and facts from which the study of Gas Dynamics will proceed.

First read Sections 1.1 through 1.5 in the text.* You may also find it useful to review your notes or textbooks used in previous Fluid Mechanics and Thermodynamics courses. Then answer the following review questions as completely as possible using <u>any</u> source you wish. Your answers are to be submitted to the instructor at which time you will be provided correct answers to these review questions. No attempt should be made to continue with this course until you feel that you understand the answers to <u>all</u> of these review questions.

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* All references to sections, equations, diagrams, etc. in the 'Text' refer to <u>GAS DYNAMICS</u> by James E. A. John. Any other referenced sources will be referred to by name.

## REVIEW QUESTIONS

1. Can you recall the Taylor's Series Expansion, it's applications and limitations?

2. How is a derivative such as $\frac{dy}{dx}$ defined?

3. Cite the basic units of measurement in the MLT system. (Metric and English)

4. What is the definition of a one pound force (in terms of pounds-mass, feet and seconds)?

5. Explain the significance of $g_c$ in Newton's Second Law.

6. What are a consistent set of units for $g_c$.

7. What is the distinguishing characteristic of a fluid (as compared to a solid)?

8. Describe the basic difference between gases and liquids. When can a gas be treated as a continuum?

9. Explain the difference between a microscopic and a macroscopic approach to the analysis of fluid behavior.

10. Describe a FLUID PROPERTY and give three or more examples of a fluid property. (Note: We will use the term 'Fluid property' to mean 'Gross' or 'Bulk' fluid property as used in the text.)

11. Fluid properties may be categorized as either INTENSIVE or EXTENSIVE. Define what is meant by each and list as many examples of each type of property as you can.

12. What does the term "specific" imply? (example: specific enthalpy)

13. What is the relationship between density and specific volume?

14. Describe an EQUATION OF STATE and recall one with which you are familiar.

15. State the PERFECT GAS LAW. When is it valid? (i.e. what are it's limitations?)

16. Give a consistent set of units for each term in the Perfect Gas Law.

17. Define a system. Describe the difference between an <u>open</u> system and a <u>closed</u> system.

18. Describe the CONTROL VOLUME APPROACH to a problem and compare it with the CONTROL MASS APPROACH. Can you identify the INDEPENDENT and DEPENDENT variables in each case?

19. Define a STATE FUNCTION and a PATH FUNCTION. Give examples of each.

20. What is a PROCESS? What is a CYCLE?

21. How is the Zero th LAW of THERMODYNAMICS related to temperature?

22. State the FIRST LAW of THERMODYNAMICS for a closed system.

23. Give a form of the FIRST LAW applicable to an open system.

24. State any form of the SECOND LAW of THERMODYNAMICS.

25. Define ENTROPY.

26. Give equations that relate entropy to
    a.) internal energy.
    b.) enthalpy.

27. Define a REVERSIBLE PROCESS for a THERMODYNAMIC system. Is any real process ever reversible? Of what practical value is the concept of reversible processes?

28. What are some IRREVERSIBLE EFFECTS?

29. What is an ADIABATIC process?

30. If a process is BOTH REVERSIBLE and ADIABATIC it is also something else. What?

31. What STATE VARIABLE is constant in an ISENTROPIC PROCESS? Is it possible to have an isentropic process that is not a reversible adiabatic process?

32. If we say $dS = 0$, does this automatically imply an ISENTROPIC situation? (where "S" is ENTROPY).

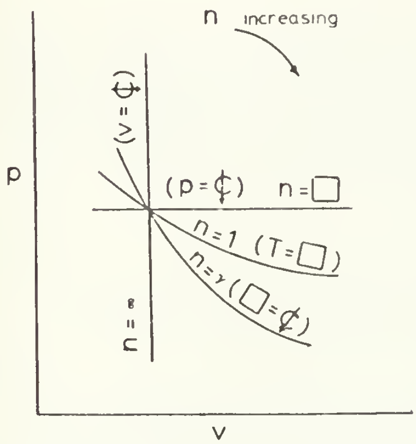
33. Define (in the form of a partial derivative) the SPECIFIC HEAT'S ($C_v$ and $C_p$). Are these expressions valid for a material in any state? Are they valid for any process?

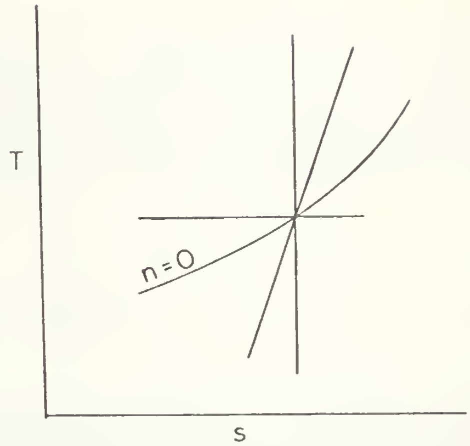
34. For a PERFECT GAS, SPECIFIC INTERNAL ENERGY is a function of which state variables?

35. For a PERFECT GAS, SPECIFIC ENTHALPY is a function of which state variables?

36. For the equation: $Q = W + \Delta E$
How are the signs conventions defined for Heat and Work?

37. <u>Process Plots</u>




$pv^n = ₵$ ; where ₵ is a "constant"
n is any positive number.

This is a General Polytropic Process.

a) In the p-v diagram place each of the following in the correct box: 0, ₵ , S.

b) In the T-s diagram, label each process line with the correct value for n and identify which fluid property is held constant.

c) Are these plots general, or are they restricted to a Perfect Gas? Why?

## COMMENTS OF THE REVIEW QUESTIONS

1. From CRC Tables (17th edition)

TAYLOR'S SERIES:

$$f(x) = f(a) + (x - a)f'(a) + \frac{(x - a)^2}{2} f''(a) + \ldots\ldots$$

The Taylor's Series expansion enables one to examine the unknown fluid properties at an arbitrary point (x) in a continuous flow field by an expansion of the derivatives of the properties at a point (a) where these are known. Note that all of the derivatives are evaluated at point (a).

2.

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

The important point here is that the derivative is a process which becomes valid in the limit as the incremental change in the state variable becomes vanishingly small.

3.

SYSTEM

| | (MKS) | Absolute Metric (CGS) | British Gravitat'nl | Absolute English | English Engineering |
|---|---|---|---|---|---|
| Mass | kilogram | gram | slug | pound-mass | pound-mass |
| Length | meter | centimeter | foot | foot | foot |
| Time | second | second | second | second | second |
| Force | newton | dyne | pound-force | poundal | pound-force |
| $g_c$ | $1 \frac{\text{kg-m}}{\text{newton-sec}^2}$ | $1 \frac{\text{gm-cm}}{\text{dyne-sec}^2}$ | $1 \frac{\text{slug-ft}}{\text{lb}_f\text{-sec}^2}$ | $1 \frac{\text{lb}_m\text{-ft}}{\text{poundal-sec}^2}$ | $32.174 \frac{\text{ft-lbm}}{\text{lb}_f\text{-sec}^2}$ |

4. A one pound force will give a one pound mass an acceleration of 32.174 ft/sec$^2$.

5. FORCE $\propto$ MASS times ACCELERATION
FORCE = (Constant)(Mass)(Acceleration)

The CONSTANT rectifies the dimensions involved. If we assign the symbol "$1/g_c$" to this constant, Newton's Law can be stated

$$F = \frac{ma}{g_c}$$

and $g_c$ will have the values indicated in the table above.

6.

$$g_c = \frac{1\ (\text{lbm})\quad 32.174\ (\text{ft/sec}^2)}{1\ (\text{lbf})} = 32.174\ \frac{\text{ft-lbm}}{\text{lbf-sec}^2}$$

7. It will continuously deform under the influence of an applied SHEARING FORCE.

8. There is no rigorous distinction between gases and liquids but generally we think of liquids as being nearly incompressible (actually they are slightly compressible) whereas gases are very compressible (thus gases assume the volume of a closed container). The problem in making a clear distinction becomes impossible when one considers a substance which is above its critical point.

9. Refer to Section 1.2 in the text (<u>Gas Dynamics</u>, John)

10. Considering the macroscopic approach of a small, but finite volume of fluid consisting of a sufficiently large number of molecules, we may define a FLUID PROPERTY by using the limit concept of two relationships such as:

$$P = \lim_{\Delta A \to a} \left( \frac{\Delta F_{normal}}{\Delta A} \right)$$

where "a" is a small but finite area.

In any case the resulting characteristic exhibited by the fluid will be as though we were considering a single point in the fluid. Such a characteristic (a property) is independent of the path by which the point was reached.
Examples are: pressure, density, temperature, entropy, enthalpy, etc.

11. Intensive properties exhibit the same characteristics independent of the amount of mass present. Temperature, pressure and specific properties are INTENSIVE. Extensive properties are dependent on the amount of mass present. Mass, energy, entropy and enthalpy are examples.

12. The term "specific" refers to the amount of an extensive property per unit mass.

13. Density is the reciprocal of specific volume:

$$\rho = \frac{1}{v}$$

14. An equation of state is a relationship among properties. The most familiar is the Perfect Gas Law:

$$P = \rho RT$$

where P is pressure
$\rho$ is density
R is a gas constant
T is temperature

15. The Perfect Gas Law is derived from kinetic theory and neglects molecular volume and inter-molecular forces. Therefore, the Perfect Gas Law can be used to deal with gases under conditions of low density (consider the variables) wherein these two phenomena are not significant factors. Gases at low pressures and high temperatures are closely described by the Perfect Gas Law.

16. P – $lbf/ft^2$ (absolute pressure)
$\rho$ – $lbm/ft^3$
T – °R (absolute temperature)
R – $\frac{ft\text{-}lbf}{lbm\text{-}°R}$

17. The term system is used in a broad sense to identify clearly whatever it is that is under consideration. A closed system is one in which mass cannot enter or leave; an open system is one in which mass may enter and/or leave.

18. The CONTROL MASS approach to a fluid flow problem utilizes a closed system, whereas the CONTROL VOLUME approach considers a fixed volume in space. Energy can cross the boundaries of either system whereas mass can only cross the boundaries of an open system.

19. A state function (also called a point function or property) is an observable characteristic which describes the physical or thermal state of a system. Two or more state functions fix the state of a system.
Path functions are transient phenomena. Consider Heat and Work:
- Systems never possess heat or work, but either or both cross the system boundary when a system undergoes a change of state.

- Both heat and work are boundary phenomena. Both are observed only at the boundaries of the system, and both represent energy crossing the boundary of the system.
- Both are PATH functions and inexact differentials. **

20. A <u>process</u> is a change of state in a system which has a unique sequence or relationship between the state variables. A <u>cycle</u> consists of several processes which vary the state of a system and return it to the original state.

21. The so called "zeroth" law of thermodynamics defines the state variable <u>temperature</u> in terms of two systems which are in thermal equilibrium with a third system.

22. For cyclic operation: $\Sigma Q = \Sigma W$ or $\oint dQ = \oint dW$

    For non-cyclic operation: $Q = W + \Delta E$
    $\delta Q = \delta W + dE$

    where heat flow into the system is positive and work transferred out of the system is positive.

    Thus a new property has been defined $\rightarrow$ E, the total energy of the system.

23. $$\frac{dQ}{dt} = \frac{dW}{dt} + \frac{\partial}{\partial t} \int_{c.v} e \rho \, dv + \int_{c.s.} e \rho \, (\overline{V} \cdot \overline{dA})$$

24. It is impossible to make any transformation whose only final result is the exchange of a non-zero amount of heat with less than two heat reservoirs and the appearance of a positive amount of work in the surroundings. (from Lord Kelvin's statement of the second law of thermodynamics, 1851)

25. $$dS = \frac{\delta Q_{REV}}{T}$$

26. For any process:

    $$Tds = du + pdv$$

    where "u" is specific internal energy, "s" is specific entropy and "v" is specific volume. T & p are temperature and pressure.

    $$Tds = dh - vdp$$

    where "h" is specific enthalpy.

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** <u>INTRODUCTION TO THERMODYNAMICS</u>, Sonntag & Van Wylen.

27. Conceptually, a reversible process is one in which both the system and its surroundings can be returned to their original conditions. Although NO real process is reversible in a strict sense, processes which closely approximate this ideal may be realized.

28. An irreversible effect causes a disturbance in the equilibrium state of a system. To quote from Elements of Gas Dynamics, by Liepmann and Roshko:

> ... A system is in equilibrium if it is free of "currents". The term current refers to the flux of a quantity like heat, mass, momentum, etc. A current of heat flows if there is a finite temperature difference; a current of mass flows if there is a difference in concentration of one component; a current of momentum flows if there is a difference in velocity.

Thus stirring a fluid, sudden heating or any process involving friction will introduce irreversibilities.

29. A process involving no transfer of heat to or from the system is called adiabatic.

30. Isentropic.

31. Entropy. It is possible to balance the increase of entropy within a system due to irreversibilities with a decrease in entropy due to heat flow and achieve a resultant change in entropy of zero (or an ISENTROPIC process). This process is obviously neither adiabatic nor reversible.
Recall: $dS = dS_e + dS_i$

32. Yes.

33.

$$c_p \equiv \left( \frac{\partial h}{\partial T} \right)_p \qquad\qquad c_v \equiv \left( \frac{\partial u}{\partial T} \right)_v$$

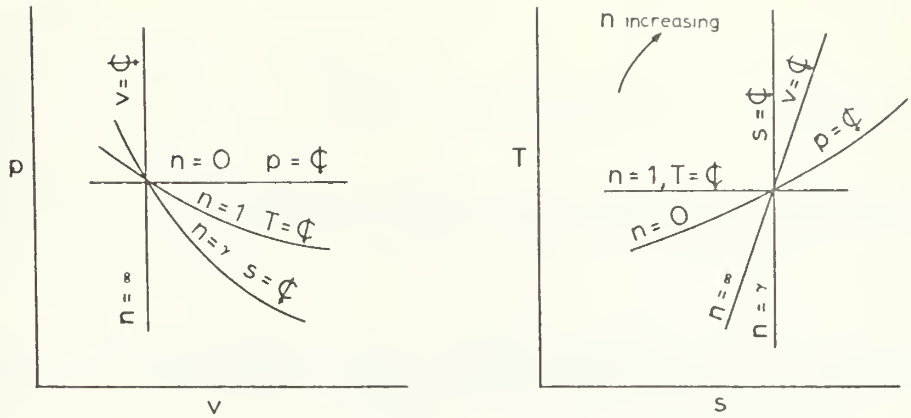
These definitions are valid for any process and for any material in any state.

34. Temperature ONLY.

35. Temperature ONLY.

36. Heat into the system is positive. Work done by the system is positive.

37.




The process lines with n = 1 referring to T = constant (₵) and n = γ referring to isentropic are restricted to Perfect Gases. Why?

## UNIT 1 - BASIC EQUATIONS - OBJECTIVES

The student shall be able to:

1. State the basic concepts from which the study of Gas Dynamics proceeds.

2. Explain the difference between a MATERIAL DERIVATIVE (or TOTAL DERIVATIVE) and a PARTIAL DERIVATIVE with respect to time.

3. Obtain the properties at one point in a flow field in terms of properties and their derivatives at another point by means of a Taylor's Series Expansion.

4. State the equation used to relate the material derivative of any extensive property to the properties inside of and crossing the boundaries of a control volume. Interpret each term in the equation.

5. Starting with the basic concepts or equations which are valid for a CONTROL MASS, obtain the integral forms of the ENERGY, CONTINUITY, and MOMENTUM equations for a CONTROL VOLUME.

6. Simplify the integral forms of the ENERGY, CONTINUITY and MOMENTUM equations for a control volume for the conditions of steady, one-dimensional flow.

7. Apply the simplified forms of the ENERGY, CONTINUITY, and MOMENTUM equations to differential control volumes.

8. Show that by introducing the concept of ENTROPY and the definition of ENTHALPY, the path function HEAT (dQ) may be removed from the ENERGY equation to yield the expression sometimes called the 'PRESSURE ENERGY Equation.'

$$\frac{dP}{\rho} + \frac{d(V^2)}{2g_c} + \frac{gdz}{g_c} + TdS_i + \delta W_s = 0$$

9. Simplify the PRESSURE ENERGY Equation to obtain BERNOULLI's Equation.

10. Apply Newton's Second Law of Motion to a differential control volume and develop the MOMENTUM equation for Steady, One-dimensional flow.

11. Identify each term and explain the restrictions and assumptions for each of the following equations:

1) $$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0$$

2) $$\delta q = d(Pv) + \delta w_{shaft} + du + \frac{d(V^2)}{2g_c} + \frac{gdz}{g_c}$$

3) $$\frac{dP}{\rho} + \frac{d(V^2)}{2g_c} + \frac{gdz}{g_c} + Tds_i + \delta w_s = 0$$

4) $$\frac{dP}{\rho} + \frac{f(V^2)}{D_E 2g_c}dx + \frac{gdz}{g_c} + \frac{VdV}{g_c} = 0$$

5) $$\frac{P_1}{\rho} + \frac{V_1^2}{2g_c} + \frac{gZ_1}{g_c} = \frac{P_2}{\rho} + \frac{V_2^2}{2g_c} + \frac{gZ_2}{g_c}$$

Study Guide Unit 1

## Basic Equations

1.1 Equations of Motion -

a) Review Section 1.4 of the Text. List the unknowns and the Equations (by name) which will be used to solve for these unknowns in the study of compressible fluid flow. How does the problem simplify if the flow is considered incompressible?

b) State in your own words what is meant by (1) the conservation of Mass, (2) the Conservation of Energy and (3) the Conservation of Momentum. Is Momentum actually conserved?

> The Basic Relationships with which we will analyze Gas Dynamics will be:
>
> (1) CONTINUITY (Derived from Conservation of Mass)
> (2) ENERGY (Derived from Conservation of Energy)
> (3) MOMENTUM (Derived from NEWTON'S SECOND LAW)
> (4) An EQUATION OF STATE | $\rho = ₵$ ; $p = \rho RT$ ; others
> (5) THERMODYNAMIC CONSIDERATIONS

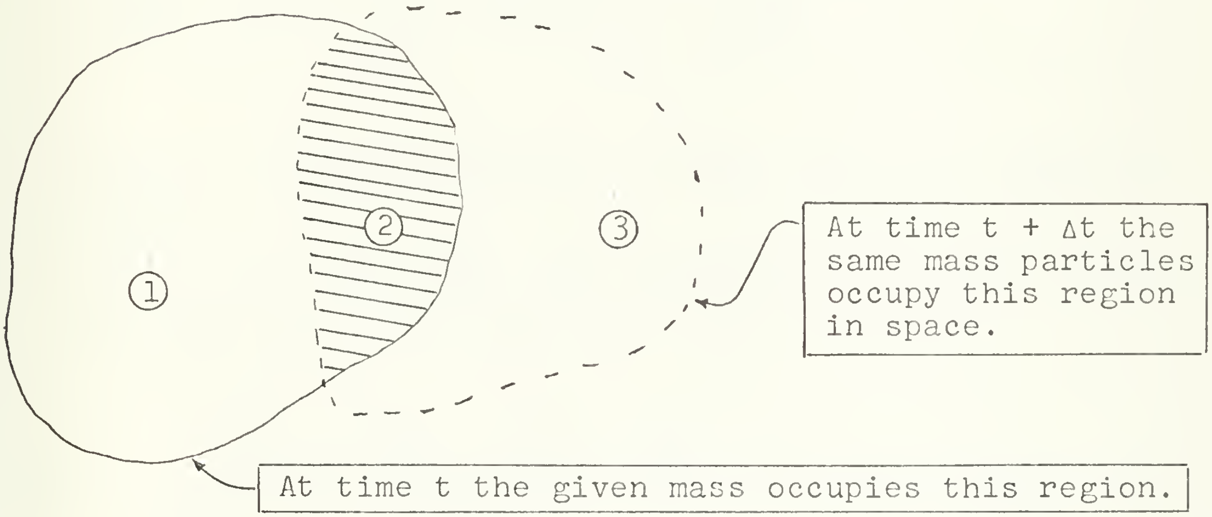
## 1.2 Transformation of a Material Derivative of any Extensive Property to a Control Volume Approach.

Let X be the total amount of any extensive property in a given mass.

Let x be the amount of X per unit mass.

Thus: $$X = \int x\,dm = \iiint \rho x\,dv = \int_V \rho x\,dv \qquad (*1.21)$$

where $dv \equiv$ incremental volume element.




Consider what happens to a material derivative such as $\frac{dX}{dt}$

$$\frac{dX}{dt} \equiv \lim_{\Delta t \to 0} \left[ \frac{(\text{Final value of } X)_{t+\Delta t} - (\text{Initial value of } X)_t}{\Delta t} \right] \qquad (*1.22)$$

Final X → X of regions 2 & 3 computed at time 't + Δt'

Initial X → X of regions 1 & 2 computed at time 't'

$$\frac{dX}{dt} \equiv \lim_{\Delta t \to 0} \frac{\left[\int_2 x\,dm + \int_3 x\,dm\right]_{t+\Delta t} - \left[\int_1 x\,dm + \int_2 x\,dm\right]_t}{\Delta t} \qquad (*1.23)$$

Consider the two terms:

$$\lim_{\Delta t \to 0} \frac{\left[\int_2 xdm\right]_{t+\Delta t} - \left[\int_2 xdm\right]_t}{\Delta t}$$

But since

$$X_2 = \int_2 xdm$$

$$\lim_{\Delta t \to 0} \left[\frac{X_2(t + \Delta t) - X_2(t)}{\Delta t}\right] = \frac{\partial X_2}{\partial t}$$

1. Partial derivative notation is used since the region is fixed.
2. As $\Delta t \to 0$, region (2) $\to$ the original confines of the mass.

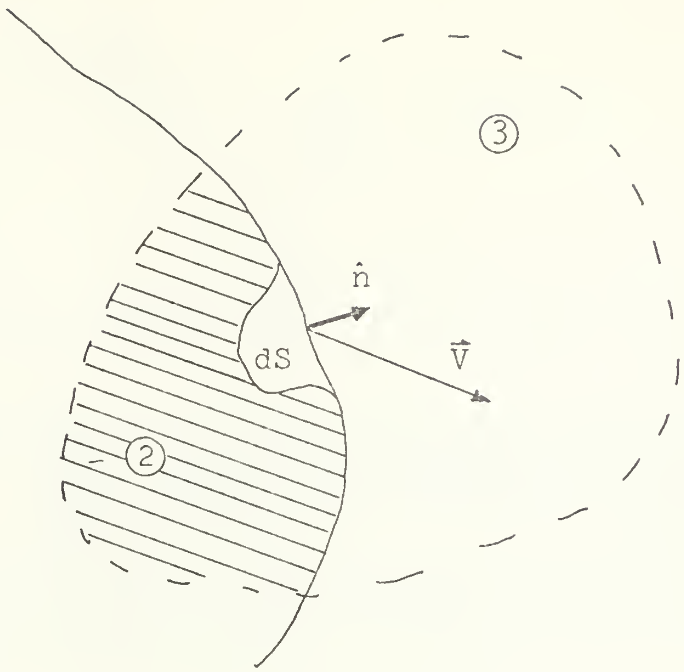
   We shall call this region the control volume.

Thus:
$$\lim_{\Delta t \to 0} \frac{\left[\int_2 xdm\right]_{t+\Delta t} - \left[\int_2 xdm\right]_t}{\Delta t} = \frac{\partial X_{c.v.}}{\partial t} = \frac{\partial}{\partial t}\int_{c.v.} \rho x dx \quad (*1.2^{l}$$

<u>Consider the term</u> $$\lim_{\Delta t \to 0^-} \frac{\left[\int_3 xdm\right]_{t+\Delta t}}{\Delta t}$$

> Note - by definition region (3) is formed by the fluid moving out of the control volume.

This integral represents the amount of X in region (3)




Let: $\hat{n}$ be a unit outward normal

| |
|---|
| Note: $\hat{n}$ ds is equivalent to $\overline{dA}$ in the Text, Section 1.6. |

ds be an increment of surface dividing regions (2) & (3)

$\vec{V}.\hat{n}$ = component of $\vec{V}$ to ds

$(\vec{V}.\hat{n})$ ds = volumetric flow rate

$\rho(\vec{V}.\hat{n})$ ds = mass flow rate

$\rho(\vec{V}.\hat{n})$ ds $\Delta t$ = amount of mass that crossed ds in time $\Delta t$

$\rho x(\vec{V}.\hat{n})$ ds $\Delta t$ = amount of X that crossed ds in time $\Delta t$

Thus $\displaystyle\int_{S_{out}} \rho x(\vec{V}.\hat{n})ds\Delta t \approx$ total amount of X in region 3 (*1.25)

and $$\lim_{\Delta t \to 0} \frac{\left[\int x\,dm\right]_{t+\Delta t}}{\Delta t} = \int_{S_{out}} x\rho(\bar{V}.\hat{n})\,ds \qquad (*1.26)$$

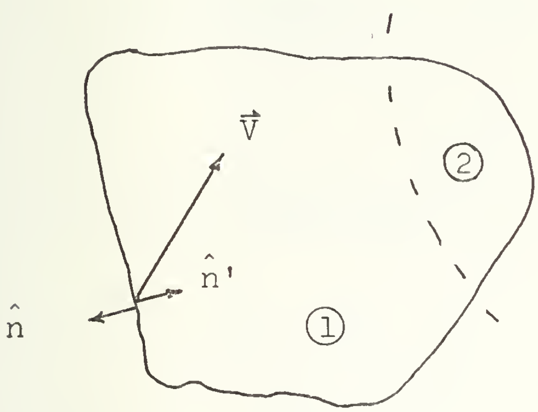
where $S_{out}$ is the surface where fluid leaves the control volume.

WHY THE LIMIT? All properties are going to be evaluated at the surface S, thus equation (*1.25) is only an approximation. It becomes exact in the limit $\Delta t \to 0$.

This integral is called a flux or rate of X flow out of the control volume.

Similarly:

with n' a unit inward normal




$$\lim_{\Delta t \to 0} \frac{\left[\int_1 x\,dm\right]_t}{\Delta t} = \int_{S_{in}} x\rho(\bar{V}.\hat{n}')ds$$

or this represents an "X Flux" into the control volume.

Note that this term appears in equation (*1.23) with a minus sign. If we use $\hat{n}$ (outward) here, since $\hat{n} = -n'$, the combination of these two terms can be obtained by integrating the following over the entire surface:

$$\int_{S_{out}} x\rho(\bar{V}.\hat{n})\,ds - \int_{S_{in}} x\rho(\bar{V}.\hat{n}')\,ds = \int_{c.s.} x\rho(\bar{V}.\hat{n})\,ds \qquad (*1.27)$$

where c.s. represents the control surface surrounding the control volume.

Final result becomes:

$$\left.\frac{dX}{dt}\right]_{\substack{\text{material}\\\text{derivative}}} = \frac{\partial}{\partial t}\underbrace{\int_{c.v.}}_{\text{Triple integral}} x\rho dv + \underbrace{\int_{c.s.}}_{\text{Double integral}} x\rho(\vec{V}.\hat{n})ds \qquad (*1.28)$$

AGAIN: This applies to any extensive (Mass dependent) property whether it is a scalar <u>or</u> a vector.

In Words: The <u>rate</u> of <u>change</u> of X for a given mass as it is moving around is equal to the rate of change of X inside the control volume <u>plus</u> the <u>net</u> flux (flow rate) of X out of the control volume. (i.e., what goes out minus what comes in).

X can be <u>momentum</u> $\rightarrow x = \vec{V}$ (used to transform Newton's 2nd law)

X can be <u>energy</u> $\rightarrow x = e = u + \frac{V^2}{2g_c} + \frac{gZ}{g_c}$

(used to transform 1st law of thermo)

X can be <u>mass itself</u> $\rightarrow x = 1$ (used to transform continuity eq.)

or other. ----

Example: This may help to sort out what is and is not a Material Derivative (or the change of an extensive property within a control mass)

For a Control Mass:

$$\frac{dQ}{dt} = \frac{dW}{dt} + \frac{dE}{dt}$$

These are <u>NOT</u> Material Derivatives as they describe the rate of transfer of energy across a <u>surface</u>.

This <u>is</u> a Material Derivative.

Thus we may summarize the two approaches to a flow problem:

1.) CONTROL MASS - wherein a given mass is observed. This is called a Closed System. (i.e., no new matter can enter or leave the system)

2.) CONTROL VOLUME - wherein a particular fixed volume in space is observed. This is called an Open System.

As an example let X (the extensive property) be Mass (M). In the transformation equation relating the Material Derivative of an extensive property (X)to the Control Volume, in Equation *1.28, X is mass, hence x is mass per unit mass, or x = 1. Thus Equation *1.28 becomes:

$$\left.\frac{dM}{dt}\right] = \frac{\partial}{\partial t} \iiint_{c.v.} \rho \, dv + \iint_{c.s.} \rho \vec{V} . d\vec{A}$$

$$\left.\frac{dM}{dt}\right]_{\substack{\text{material} \\ \text{derivative}}} = 0$$

so ...

$$0 = \frac{\partial}{\partial t} \iiint_{c.v.} \rho \, dv + \iint_{c.s.} \rho \vec{V} . d\vec{A}$$

This is the CONTINUITY EQUATION for a control volume. There are other forms of this equation for special conditions such as steady flow, etc. which we will now develop.

## 1.3 Control Mass versus Control Volume

| |
|---|
| COMMENT - The concept developed in 1.2 is presented in an abbreviated form by the Text. Compare these two parallel developments as this important concept must be thoroughly understood. |

a) Read section 1.6 of the Basic text.

b) Can you interpret the conversion from the consideration of a fixed MASS system and the properties associated with it to a CONTROL VOLUME approach? Which is more convenient and WHY?

c) Do you understand the significance of each term in Equation (1.7)?[1] What does each term mean (Explain in words).

d) Show which term will drop out of Equation (1.7) in any steady flow situation and explain why.

---

Note 1: Equation numbers without an asterisk refer to equations in the text (JOHN).

Equations with an asterisk (e.g., *1.23) refer to equations in the study guides.

## 1.4 CONSERVATION OF MASS

a.) Read Section 1.7 in the text.

| |
|---|
| COMMENT - In this course we shall assume that Matter and Energy are each separately conserved, as we shall not consider transformations such as occur in nuclear reactions. |

b.) What is the Principal of Conservation of Mass, in words?

c.) Now skip ahead and read Section 1.12. The real importance of the One-Dimensional Flow assumption is that it permits us to easily obtain approximate solutions (with acceptable engineering accuracy) to problems whose exact solutions would be extremely difficult. Any problem solution based on a simplifying assumption will yield results which vary to some extent from the real-world problem. The decision to utilize a simplification depends on its correlation to experimental results, the availability and cost of alternate methods of solution and the accuracy required of the solution.

d.) Now return to pg. 8 of the text and determine in detail the restrictions imposed in progressing from Eq. (1.9) to Eq. (1.10).

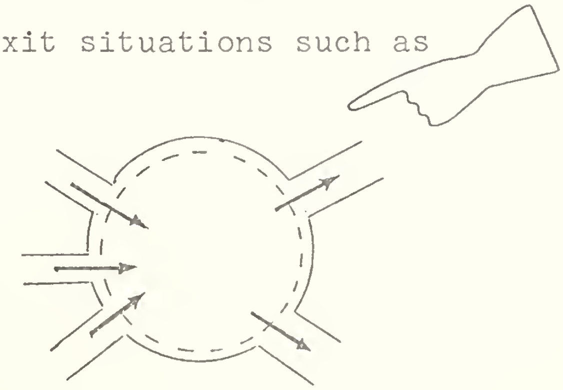
NOTE WELL: The condition of ONE-DIMENSIONAL Flow imposes NO restrictions on density, area, velocity, (or any other fluid property) from section to section in the direction of flow, but does say that for a given location at a given instant of time, the flow variables are assumed constant over the entire cross-section.

e.) Equation (1.10) is often written $\rho AV$ = constant, in which case the V is assumed to be the component of velocity perpendicular to the area. What quantity is the constant?

HINT: Work out the Units

f.) Is it correct to say that the continuity equation can be stated as "The flow rate into a control volume equals the flow rate out of the control volume?" Is this mass flow rate or volumetric flow rate? What condition must exist before the above statement is true? Follow through examples 1.1, 1.2. Can you LIST the assumptions made for each of these problems?

g.) It is more convenient for the case of multiple entry and exit situations such as



to express the continuity equation for one-dimensional, steady flow as: $\sum \rho AV = 0$

h.) An alternate form of the continuity equation (for one-dimensional steady flow) is obtained by first taking the natural logarithm of [$\rho AV$ = constant] and then differentiating the result. Try it! You should get:

$$\frac{d\rho}{\rho} + \frac{dV}{V} + \frac{dA}{A} = 0 \qquad (*1.41)$$

This form is useful in interpreting the changes that must occur as fluid flows through a duct, channel or stream tube. It indicates that if mass is to be conserved, the changes in density, velocity and cross-sectional area must compensate for one another. For example, if the area is constant ($dA = 0$), then any increase in velocity must be accompanied by a corresponding decrease in density. This form of the continuity equation will also be useful in future derivations.

## 1.5 Conservation of Energy

a.) Read Section 1.9 of the text.

Section 1.8 will be considered after the ENERGY concepts are firm.

b.) Do you recall the essential difference between a PATH FUNCTION and a STATE FUNCTION? The key lies in the definition of PATH functions which are only defined in crossing a system boundary. "Heat" and "pressure" are examples respectively of a PATH and a STATE function. List as many of each as you can recall.

| PATH FUNCTIONS | STATE FUNCTIONS |
| --- | --- |

c.) What notation is used to represent the infinitesimal change in a PATH function by JOHN? How about the notation for an infinitesimal change in a STATE FUNCTION? Why the difference?

Example: $\delta Q = \delta W + dE$

Note that in equations 1.16 and subsequent the expression dQ/dt uses the same differential symbol for Heat (Q), a PATH function and for Energy (E) a STATE function. This is valid because we are now dealing with a control volume, so the change in Heat (or work) is now a function of time only.

d.) Two important concepts included in the development of equation (1.19) are:

1) FLOW WORK

2) ENTHALPY

Flow work is the work done to push the fluid into (or out of) the system (i.e., CONTROL VOLUME). Since it is often difficult to measure Flow work being done on the system (whereas the Shaft work and other types of work can usually be measured), it is advantageous to combine Flow work with specific Internal energy (a thermodynamic property) and thus define ENTHALPY (h).

e.) The formulation of the general ENERGY Equation as expressed in equation (1.19) can often be simplified to facilitate obtaining a solution to practical problems in engineering. Consider the case of STEADY FLOW. We can summarize the effects of STEADY FLOW by two conditions:

1) MASS FLOW IN equals the MASS FLOW OUT of the CONTROL VOLUME. Why is this so? Does this define steady flow?

2) $\frac{\partial(\text{anything})}{\partial t} = 0$

| Note carefully the use of partial versus the ordinary derivative |
|---|

This assumption causes the first term of eq. 1.19 to vanish. Similarly, as in examples 1.5 and 1.6, real problems can be readily treated by judicious definition of the control volume, which makes the surface integral in eq. (1.19) extremely easy to evaluate.

f.) The general ENERGY equation may be viewed as analogous to a bank account. Each of the terms represents a certain type of transaction, which must be balanced against each other to balance the books in accordance with the Law of Conservation of ENERGY. To consolidate your understanding of the effect of each term, next to each of the statements below, write the term from the GENERAL ENERGY EQUATION, Equation (1.19), whose function is described. (→ You may have to separate certain terms)

1) The time rate of change of total energy within the control volume ____________________

2) The net efflux of KINETIC ENERGY ______________________________

3) The net efflux of POTENTIAL ENERGY associated with the earth's gravitational field. ____________________

4) The net efflux of flow work and internal energy ____________________

5) The rate at which heat enters the control volume ____________________

6) The rate at which work (other than flow work) leaves the control volume ____________________

g) From Thermodynamic considerations entropy can be broken down as follows:

$$ds \equiv ds_e + ds_i \qquad (*1.51)$$

where: "$ds_e$" represents entropy change caused by External Heat Transfer (or HEAT crossing the boundary). It is assumed that this Heat is transferred Reversibly.

"$ds_i$" represents the Entropy change caused by all the Internal IRREVERSIBLE effects.

THUS 1) $ds_e = \frac{\delta Q}{T}$ (This can be + or - depending on the sign of $\delta Q$)

2) $ds_i$ is always positive!

Now recall the "INEQUALITY of CLAUSIUS." Develop this from *1.51.

$$\oint \frac{\delta Q}{T} \leq 0 \qquad (*1.52)$$

From your Review Questions you can recall

$ds_e = 0$ implies ADIABATIC

$ds_i = 0$ implies REVERSIBLE

and $ds = 0$ means ISENTROPIC, therefore if a situation both ADIABATIC and Reversible it is ISENTROPIC.

h) To simplify the application of the energy equation (1.19) to many engineering applications we will again apply the assumption of ONE-DIMENSIONAL FLOW.

Now for ONE-DIMENSIONAL steady flow a differential control volume may be conceptualized as the dotted area in Figure *1.51.

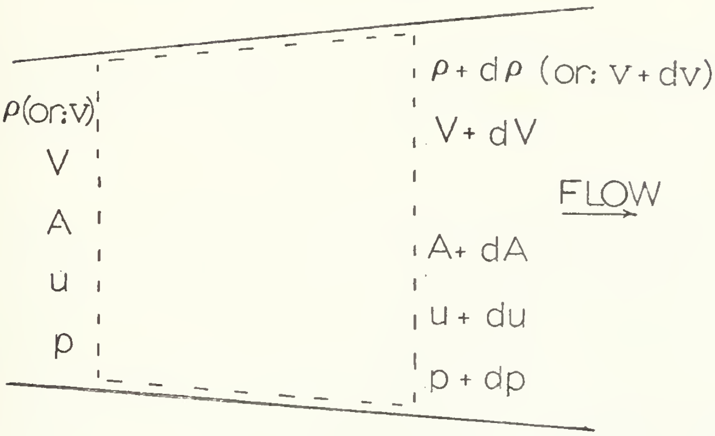



Figure *1.51

Thus we may apply the Energy equation (1.19) to this control volume, and divide out the flow rate with the following result:

$$\delta q = [(p+dp)(v+dv) - pv] + \delta w_{shaft} + [(u+du) + \frac{(v+dV)^2}{2g_c} + \frac{g(z+dz)}{g_c}] - [u + \frac{V}{2g_c} + \frac{gz}{g_c}] \qquad (*1.53)$$

If we expand this and neglect terms of higher than linear order we now have the Energy Equation in differential form:

$$\delta q = d(pv) + \delta w_{shaft} + du + \frac{d(V^2)}{2g_c} + \frac{gdz}{g_c} \qquad (*1.54)$$

Fill in the intermediate steps for exercise.

How would you define $\delta q$ and $\delta w$?

i) The stage is now set to develop the "PRESSURE ENERGY EQUATION." From the Thermodynamic relation

$$T\,ds = dh - v\,dp \qquad (*1.55)$$

we can now substitute Equation *1.51 to obtain

$$dh = T ds_e + T ds_i + \frac{dp}{\rho} \qquad (*1.56)$$

Now by substituting from equation *1.56 for dh in place of $du + d(Pv)$ in equation *1.54 and recognizing that $T ds_e = \delta q$, we can obtain the useful form of the energy equation often called the PRESSURE ENERGY equation.

$$\frac{dp}{\rho} + \frac{d(V^2)}{2g_c} + \frac{g\,dz}{g_c} + T ds_i + \delta w_{shaft} = 0 \qquad (*1.57)$$

j) The PRESSURE ENERGY EQUATION is the source of a familiar relation from FLUID MECHANICS. If we consider the flow as REVERSIBLE ($ds_i = 0$) and No shaft work crosses the boundary ($\delta w_{shaft} = 0$) and if the fluid is considered incompressible ($\rho = \text{¢}$) then we can integrate the Pressure Energy Equation between two points and obtain BERNOULLI's Equation.

$$\frac{p_1}{\rho} + \frac{V_1^2}{2g_c} + \frac{gz_1}{g_c} = \frac{p_2}{\rho} + \frac{V_2^2}{2g_c} + \frac{gz_2}{g_c} \qquad (*1.58)$$

Summarize the assumptions made for Bernoulli's Equation for your own review.

1. ______________________________

2. ______________________________

3. ______________________________

4. ______________________________

5. ______________________________

## 1.6 - MOMENTUM

a. Prior to the development of the equation(s) describing momentum, consider the concept of momentum – could you describe momentum to a 'non-engineering type'? Reflect on your own understanding and if you feel uncertain about this concept, break out your Physics text and refresh your understanding of 'MOMENTUM' and 'IMPULSE'.

b. Study Section 1.8 in the text, including examples 1.3 and 1.4. Are we considering angular momentum? How about acceleration effects?

c. Since neither angular momentum nor any acceleration effects are considered, we find that, from the text, equation (1.13) is in reality valid only under quite restricted conditions. Explain in your own words the restriction related to each of the below listed concepts.

   a) Inertial Reference
   b) No Angular Momentum
   c) Control Volume (Fixed or Constant Velocity)

   Can you take equation (1.13) and "cross" both sides with the radius and come up with a similar relationship to deal with angular momentum?

d. In Section 1.8 of the text, the material derivative of the extensive property momentum is being transformed to a control volume consideration, which is then related to the forces acting on the control volume by Newton's Second Law of Motion. We can apply to equation (1.13) in the text, the simplifying assumption of Steady and

One-dimensional flow. For One-dimensional flow, the second term on the right side

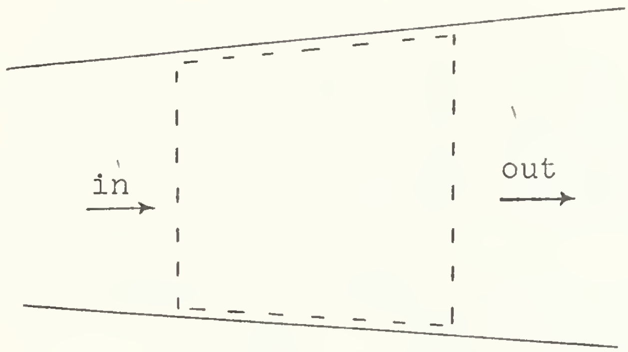



Figure *1.6-1

which describes the net amount of momentum per unit mass crossing the control boundary simplifies as follows:

$$\iint_{c.s.} \frac{\vec{V}}{g_c} \rho(\vec{V}.\vec{dA}) = \left.\frac{\dot{m}\vec{V}}{g_c}\right|_{out} - \left.\frac{\dot{m}\vec{V}}{g_c}\right|_{in} \qquad (*1.6,1)$$

From Section 1.5 we can recall the two useful effects of Steady Flow:

$$\frac{\partial}{\partial t} \text{(anything)} = 0$$

and: $\sum$ Mass flow in $=$ $\sum$ Mass flow out $= ₵ = \dot{m}$

Therefore Equation (1.13) becomes

$$\sum \vec{F}_{\text{on fluid}} = \frac{\dot{m}}{g_c} (\vec{V}_{out} - \vec{V}_{in}) \qquad (*1.6,2)$$

e. Having obtained a relation which is now expressed in terms of the mass flow rate and velocities in and out of the control volume, we need now to examine the Forces acting on the control volume.

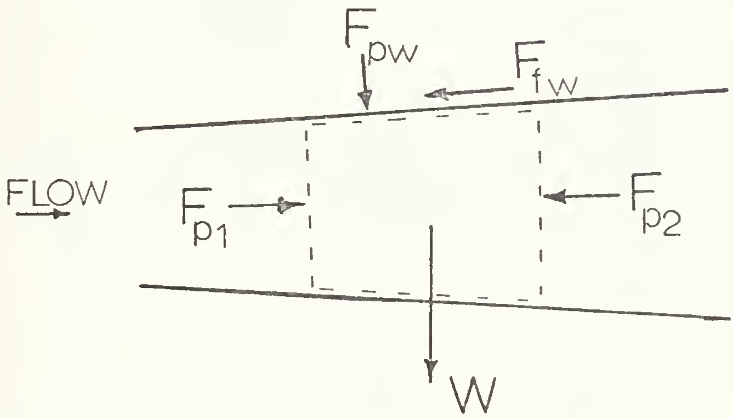



Figure *1.6-2

In Figure *1.6-2 we see summarized the various forces acting on the control volume, where:

- $W$ = Weight of Fluid in the control volume
- $F_{p_1}$ = Upstream pressure forces
- $F_{p_2}$ = Downstream pressure forces
- $F_{p_w}$ = Pressure forces exerted by the walls
- $F_{f_w}$ = Friction forces exerted by the wall

It is often common practice to group the last two together and refer to them as 'Enclosure Forces'. Where necessary

we can also include other forces such as electrical, magnetic or other types of field forces which for simplicity we will not consider here.

We establish co-ordinates and designate the various forces by subscript acting on a differential control volume of length dx.

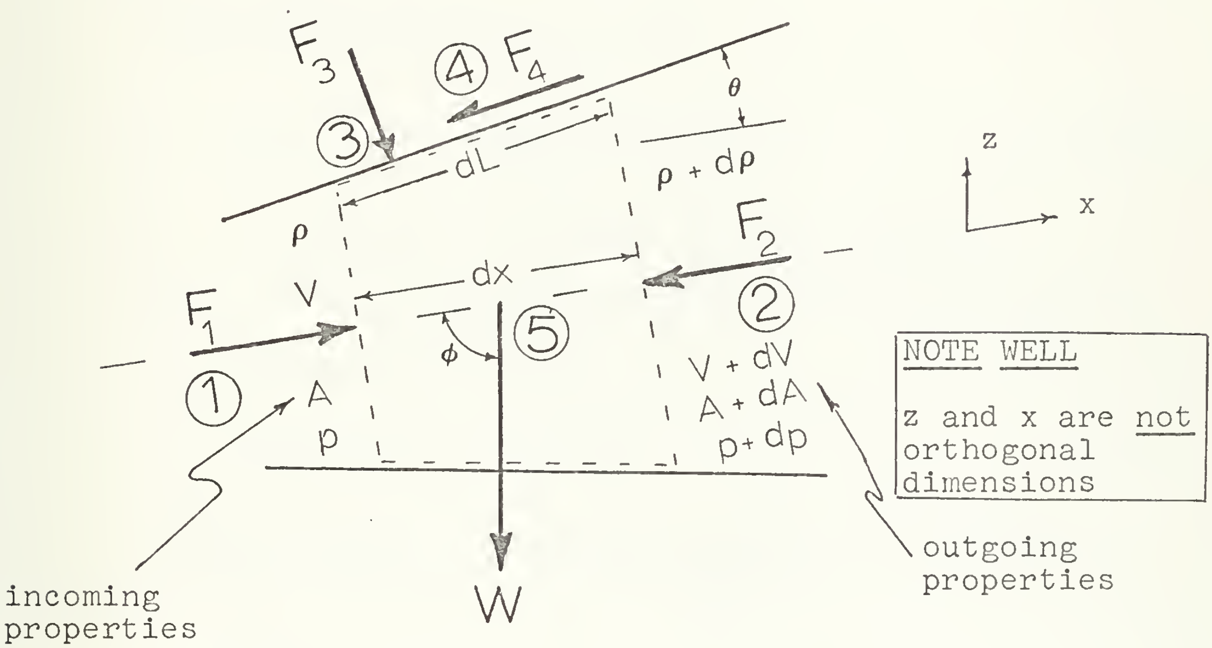



Figure *1.6-3

Since this is steady, ONE-DIMENSIONAL flow in the x-direction, the sum of Forces in any other direction must equal zero from equation (*1.6-2).

f. We can now summarize the x-components of the Forces. Note we will establish the sign convention that force components in the upstream direction are positive.

Upstream Pressure Forces (1)

$$F_{1x} = PA \qquad (*1.6,3)$$

Downstream Pressure Forces (2)

$$F_{2x} = -(P + dP)(A + dA) \qquad (*1.6,4)$$

Expanding and Neglecting higher ordered terms:

$$F_{2x} = -[PA + PdA + AdP] \qquad (*1.6,5)$$

Wall Pressure Forces (3)

$$F_{3x} = (p + \frac{dp}{2})(\mathcal{P}_f dL) \sin\theta \qquad (*1.6,6)$$

where $\mathcal{P}_f$ is the Mean Wetted Perimeter

and $\theta$ is the divergence angle of the wall

if we recognize that

$$(\mathcal{P}_f dL) \sin\theta = dA \qquad (*1.6,7)$$

and we expand and neglect higher ordered terms.

$$F_{3x} = PdA \qquad (*1.6,8)$$

Friction Forces (4)

If we define $\tau_f$ as the mean shear stress along the wall

$$F_{4x} = -[\tau_f (\mathcal{P}_f dL) \cos\theta] \qquad (*1.6,9)$$

recognizing that $dL \cos\theta = dx$

$$F_{4x} = -\tau_f \mathcal{P}_f \, dx \qquad (*1.6,10)$$

Gravity Force (5)

$$F_{5x} = - [ (A + \frac{dA}{2}) dx ] [\rho + \frac{d\rho}{2}) ] \frac{g}{g_c} \cos \phi \quad (*1.6,11)$$

where $\phi$ is the angle of inclination of the x axis but $dx \cos \phi = dz$. Expanding and neglecting higher order terms:

$$F_{5x} = - A\rho \frac{g}{g_c} dz \qquad (*1.6,12)$$

g. Having shown that the right side of (*1.6,2) is:

$$\frac{\dot{m}}{g_c} [V_{out} - V_{in}] \qquad (*1.6,2a)$$

The x component of this becomes:

$$\frac{\dot{m}}{g_c} [V + dV - V] \qquad (*1.6,2b)$$

or

$$\frac{\dot{m}}{g_c} dV$$

You should equate the forces to the right side, cancel terms and obtain the following result:

$$AdP + \tau_f \, \wp \, dx + \rho A \frac{g}{g_c} dz + \rho \frac{AVdV}{g_c} = 0 \qquad (*1.6,13)$$

To evaluate the <u>Shear</u> term we will introduce a non-dimensional friction co-efficient (f) which will relate

shear force to the dynamic pressure. (Sometimes the friction factor is defined without the factor of 4.)

$$f \equiv \frac{4\tau_f}{\frac{\rho V^2}{2g_c}} \qquad (*1.6,14)$$

We also introduce the concept of an 'Equivalent diameter'.

$$D_E \equiv \frac{4A}{\wp} \qquad (*1.6,15)$$

Then

$$\wp = \frac{4A}{D_E} \qquad (*1.6,15a)$$

and substituting these expressions into Equation (*1.6,13), divide the equation by density and we obtain a useful form of the <u>Momentum Equation</u> in the direction of fluid flow for steady, one-dimensional flow.

$$\frac{dP}{\rho} + \frac{fV^2}{D_E 2g_c}\,dx + \frac{g}{g_c}\,dz + \frac{VdV}{g_c} = 0 \qquad (*1.6,16)$$

## 1.7 - THERMODYNAMIC CONSIDERATIONS and EQUATION of STATE

a. Read Section 1.10. From your basic Thermodynamics course you may recall that entropy changes can fall into one of two different categories:

$$ds = ds_i + ds_e$$

where $ds_i$ is the change in entropy due to irreversible effects (such as friction)

and $ds_e$ ( $\frac{dQ}{T}$ ) is the change in entropy due to heat transfer.

> <u>Comment</u> on $ds_i$
>
> $ds_i$ is always POSITIVE since all irreversible effects generate entropy. But if, in a process, the changes in pressure temperature, etc. are vanishingly small, then this process may be treated as reversible, hence $ds_i = 0$.

> <u>Comment</u> on $ds_e$
>
> $ds_e$ can be positive or negative (depending on the direction of the heat transfer) and is zero if no heat transfer occurs.

b. Read Section 1.11. This section reviews some vital concepts and relations which will be vital in many applications. Example 1.7 provides an excellent opportunity to follow through a problem using a systematic problem solution technique.

A recommended method for approaching a problem is provided below. Although each of these factors may not figure in every problem, as was the case in example 1.7, they should help you organize a problem solution.

1) Identify (and sketch) CONTROL VOLUME, the dotted area in Example 1.7.

2) Identify FORCES acting on the matter inside the CONTROL VOLUME.

3) Identify where the fluid enters and leaves the CONTROL VOLUME.

4) Identify any energy (Q and W) being transferred across the control surface.

5) ESTABLISH A CO-ORDINATE SYSTEM. DON'T FORGET SIGN CONVENTIONS.

6) BE ESPECIALLY CAREFUL WITH THE SIGNS OF VECTOR QUANTITIES SUCH AS $\vec{V}$ and $\vec{F}$.

7) EVALUATE WHAT IS KNOWN and WHAT IS UNKNOWN. DECIDE WHAT RELATIONSHIPS WILL BE USEFUL.

| |
|---|
| The Basic Relationships with which we will analyze Gas Dynamics will be: |
| 1) CONTINUITY (Derived from Conservation of Mass) |
| 2) ENERGY (Derived from Conservation of Energy) |
| 3) MOMENTUM (Derived from NEWTON'S SECOND LAW) |
| 4) An EQUATION of STATE $\rho = ¢$ <br> $p = \rho RT$ <br> others |
| 5) THERMODYNAMIC CONSIDERATIONS |

c. Review Section 1.13.

## UNIT 2 - STAGNATION CONCEPTS WAVE PROPAGATION and MACH NUMBERS

### - OBJECTIVES -

After successfully completing this unit you should be able to:

1. Explain the stagnation state concept, how it is achieved and utilized.

2. Compare the concept of a STAGNATION property with that of a STATIC property.

3. Draw a T - S diagram representing a flow system and indicate static and stagnation points for an arbitrary section.

4. Define STAGNATION ENTHALPY.

5. Introduce the stagnation concept into the "Pressure Energy" equation and derive the "Stagnation Pressure Energy" equation.

6. Explain how sound is propagated through any medium (solid, liquid or gas).

7. Define SONIC VELOCITY.

8. Utilize a control volume analysis to derive the general expression for the velocity of wave propagation in an arbitrary medium, starting with the "Continuity" and "Momentum" equations for steady, one dimensional flow.

9. Recall the relations for:
   a. Speed of Sound in an arbitrary medium.
   b. Speed of Sound in a Perfect Gas.
   c. Mach Number.

10. Discuss the propagation of signal waves from a moving body in a fluid, and explain what is meant by 'Zone of Action', 'Zone of Silence', 'Mach Cone', and 'Mach Angle'.

11. Graphically portray the simplified flow over a wedge shaped body for SUBSONIC and SUPERSONIC flow.

12. Express the basic equations in terms of Mach number for a Perfect Gas by showing that:

    a. the "Continuity" equation can be written as

$$\dot{m} = PAM\sqrt{\frac{\gamma g_c}{RT}}$$

b. the "Energy" equation can be written as

$$\left(1 + \frac{\gamma-1}{2} M^2\right)dh + \frac{\gamma-1}{2} h dM^2 = \delta q - \delta w_s$$

c. the "Momentum" equation can be written as

$$\frac{dP}{P} + \frac{\gamma}{2} dM^2 + \frac{\gamma}{2} M^2 \frac{dT}{T} + \frac{\gamma}{2} M \frac{fdx}{D_E} = 0$$

UNIT 2 - STUDY GUIDE

## 2.1 The Stagnation Concept

a. We introduce the concept of a reference state defined as that thermodynamic state which would exist if the fluid were brought to zero velocity and zero potential. To yield a consistent reference state we must qualify how this "stagnation process" would be accomplished. The stagnation state must be reached:

(a) without any energy exchange $(Q = W = 0)$

(b) without losses.

By virtue of (a), $dS_e = 0$, and from (b), $dS_i = 0$. Thus the stagnation process is isentropic.

b. We can imagine the following example of actually carrying out the stagnation process. Consider fluid which is flowing under condition 1 in Figure *2.1 (these conditions are referred to as the "static" conditions). At (2) the fluid has been brought to zero velocity and zero potential under the above restrictions.

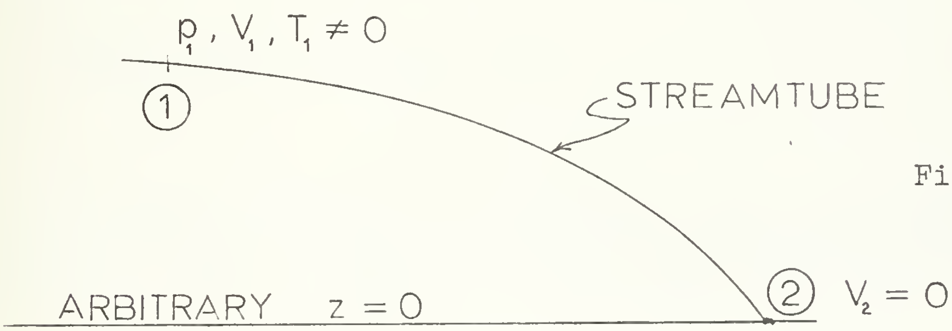



Figure *2.1

c. Apply the "energy" equation to the streamtube and you have:

$$h_1 + \frac{V_1^2}{2g_c} + \frac{gz_1}{g_c} + \cancelto{0}{Q_{1-2}} = h_2 + \cancelto{0}{\frac{V_2^2}{2g_c}} + \cancelto{0}{\frac{gz_2}{g_c}} + \cancelto{0}{W_{1-2}} \qquad (*2.1,1)$$

which simplifies to:

$$h_1 + \frac{V_1^2}{2g_c} + \frac{gz_1}{g_c} = h_2 \qquad (*2.1,2)$$

But condition (2) represents the "stagnation state" corresponding to the "static state" (1). We thus call $h_2$ the stagnation or total enthalpy corresponding to state (1) and designate it as $h_{t_1}$.

Thus:

$$h_{t_1} = h_1 + \frac{V_1^{\ 2}}{2g_c} + \frac{gz_1}{g_c} \qquad (*2.1,3)$$

Or, for any static state we have in general:

$$h_t = h + \frac{V^2}{2g_c} + \frac{gz}{g_c} \qquad (*2.1,5)$$

d. The introduction of the stagnation (or total) enthalpy makes it possible to write equations in a simplified form. For example, the steady flow energy equation becomes:

$$dq = dw_s + dh_t \qquad (*2.1,6)$$

or

$$h_{t_1} + q_{1-2} = h_{t_2} + w_{s_{1-2}} \qquad (*2.1,7)$$

e. You should note that the stagnation state is a reference state and may or may not actually exist in the flow system. Also, in general, each point in a flow system has a different stagnation state as shown below:

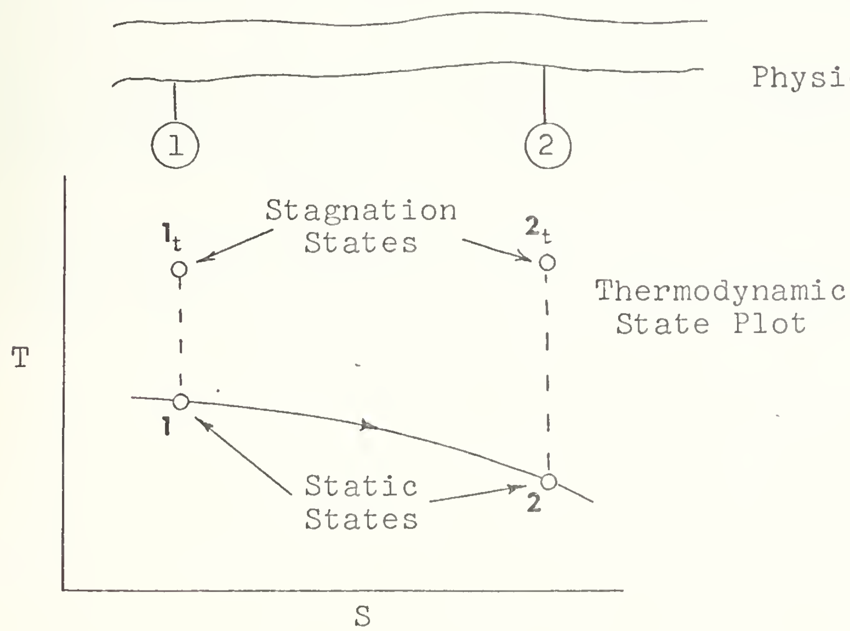



Figure *2.2

Also one must realize that when the frame of reference is changed then stagnation conditions change, although the static conditions remain the same. (Static properties are defined as those that would be measured if the measuring devices move with the fluid.)

Consider still air and the earth as a reference frame.

$V = 0$ $P = 14.7$ psia $T = 520°R$

$P_t = 14.7$ psia $T_t = 520°R$

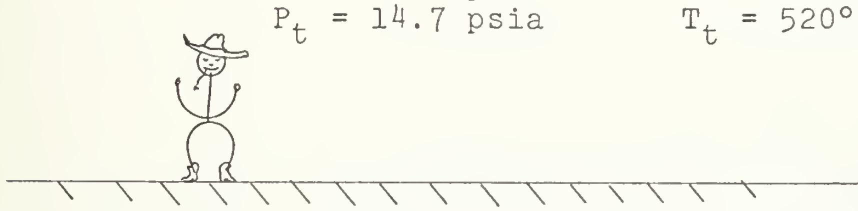


Figure *2.3

In this case, since the velocity is zero (with respect to the frame of reference), the static and stagnation conditions are the same.

f. Now let's change the frame of reference by flying through the air on a missile at 600 ft/sec. As we look forward it appears that the air is coming at us at 600 ft/sec. Its static pressure and temperature are unchanged at 14.7 psia and 520° R respectively.

$V = 600$ ft/sec

$P = 14.7$ psia $\neq P_t$

$T = 520°$ R $\neq T_t$

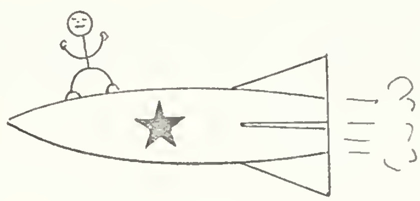


Figure *2.4

But now the air has a velocity (with respect to the frame of reference) and thus the static and stagnation conditions are different.

g. You will soon learn how to compute the stagnation conditions. Incidentally, is there any place in this last system where the stagnation conditions <u>actually</u> exist?

2.2 The Stagnation Pressure Energy Equation

a. Consider two stations in a flow system that are very close together and with thermodynamic states differentially separated as shown in Figure *2.5. Also shown are the corresponding stagnation states for these two stations.

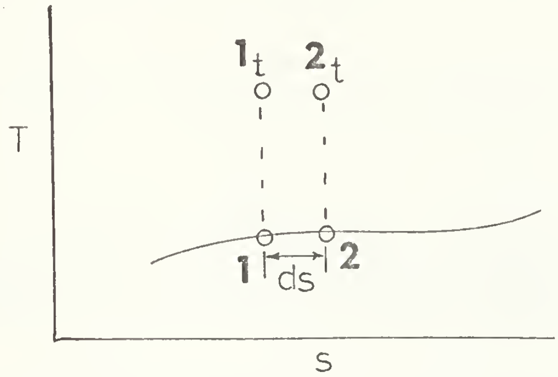



Figure *2.5

We may write the following property relation between points 1 and 2:

$$Tds = dh - vdp \qquad (*2.2,1)$$

Since this is a valid property relation we may also write this between points $1_t$ and $2_t$.

$$T_t ds_t = dh_t - v_t dp_t \qquad (*2.2,2)$$

However, $ds_t = ds$

and $ds = ds_e + ds_i$

Thus we may write:

$$T_t(ds_e + ds_i) = dh_t - v_t dp_t \qquad (*2.2,3)$$

Recall the "energy" equation written in the form:

$$\delta q = \delta w_s + dh_t \qquad (*2.2,4)$$

Substitute equation *2.2,3 into equation *2.2,4 and you should obtain:

$$\delta q \;=\; \delta w_s + T_t(ds_e + ds_i) + v_t dp_t \qquad (*2.2,5)$$

Also recall that: $\delta q = Tds_e$ (*2.2,6)

By substituting equation *2.2,6 into *2.2,5 and noting that –

$$v_t = \frac{1}{\rho_t}$$

you should obtain the following equation which is called the "Stagnation Pressure Energy" equation:

$$\frac{dP_t}{\rho_t} + \delta w_s + ds_e(T_t - T) + T_t ds_i \;=\; 0 \qquad (*2.2,7)$$

b. Consider what happens when

(a) There is no work transfer $\delta w_s = 0$

(b) There is no heat transfer $ds_e = 0$

(c) There are no losses $ds_i = 0$

Under these conditions $\dfrac{dP_t}{\rho t} = 0$

or $dP_t = 0$ (*2.2,8)

or $P_t$ = constant (*2.2,9)

Note that in general the total pressure will <u>not</u> remain constant; only under a special set of circumstances will equation *2.2,9 hold true.

## 2.3 Velocity of Sound

a. Read Sections 2.1, 2.2 and 2.3 in the text.

b. Recall from your previous work in fluid mechanics, the definition of a stream line. What, if any, properties reamin constant along a stream line? What significance does the distance between stream lines have in an incompressible medium?

c. Consider carefully the implications of section 2.2 with regard to the velocity of sound in a compressible medium. It is important to note that a sound wave results from an infinitesimally small impulse. The text uses a moving control volume which encloses the wave to analyze this unsteady situation as a steady flow problem. We can view this process as a simple change of reference from that of a fixed observer viewing the moving wave to that of an observer who is "riding" the wave. The moving observer experiences the fluid flowing from right to left in Figure 2.5 at a velocity of sound (a) upstream and velocity (V-a) downstream. This does not change the static conditions. What about the stagnation conditions?

d. Follow through Example 2.1. The author of our text introduces the relationship of 'compressibilities' which may be unfamiliar to you. What other ratio is expressed by ($\gamma$)? We need not concern ourselves at this stage with situations in which air does not behave approximately as a perfect gas, other than to examine each problem with this in mind. In general, for flow situations where extremely high pressures and/or extremely low temperatures are encountered the 'Perfect Gas' approximation is not valid. Otherwise we may treat air and other gasses (if no appreciable molecular interaction is taking place) as a perfect gas.

e. Read sections 2.4 and 2.5 in the text. In Figure 2.10 in the text, the circles indicate the location of the original sound wave emitted while at point '0'. Figure *2.6 in the Study Guide shows the location of the sound wave initially emitted at times, t = 0, 1, & 2 as they appear at time, t = 3.

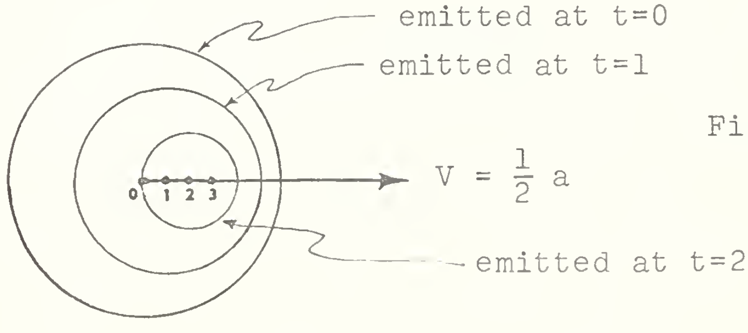



Figure *2.6

f. Notice the distance between successive sound waves is less in the direction of travel of the point projectile. Now compare Figure *2.6 to Figure 2.11 in the text. In Figure 2.11 the point is overtaking the sound waves it emits as it moves. The waves are left behind creating a situation in a sense like the wave created by dragging the point of a stick rapidly across the surface of a still body of water. The Mach Wave (or Mach Cone) produced by a point source is an infinitely weak pressure wave, but the addition of a multitude of such waves produced by a finite body results in the shock shown in Figure 2.13 in the text.

g. It should be emphasized again that both velocity and speed of sound are local conditions.

Now consider the problem below:


Note: No information may be assumed about-pressure, area, temperature, etc.-changes from 1 to 2.

$V_1$ = 500 ft/sec  $V_2$ = 1000 ft/sec

$M_1$ = .5  $M_2$ = ?

Therefore:

$a_1$ = 1000 ft/sec  $a_2$ = ?

Is enough information given to compute $a_2$?

## 2.4 - Basic Equations in Terms of Mach Number

a. The importance of Mach Number as a parameter which controls the behavior of fluid flow suggests the value of expressing the basic equations developed in Unit 1 in terms of Mach Number as well as velocity. This can be done for steady, one dimensional flow quite readily, for the case of perfect gases.

b. For the CONTINUITY equation --

Equation (1.10) from the text may be expressed for steady flow as:

$$\dot{m} = \rho AV = ¢ \qquad (*2.4,1)$$

where $(\dot{m})$ is mass flow rate.

From the Perfect Gas Law:

$$\rho = \frac{P}{RT} \qquad (*2.4,2)$$

and the definition of Mach Number:

$$V = Ma \qquad (*2.4,3)$$

Now recall the expression for sonic velocity in a Perfect Gas:

$$a = \sqrt{\gamma g_c RT} \qquad (*2.4,4)$$

where $\gamma$, $g_c$ and R are defined in Section 1.11 of the text.

We thus have:

$$\dot{m} = \rho AV = \frac{PAM}{RT}\sqrt{\gamma g_c RT} = PAM\sqrt{\frac{\gamma g_c}{RT}} \qquad (*2.4,5)$$

Thus, for the steady, one dimensional flow of a Perfect Gas, the "Continuity" equation becomes:

$$PAM\sqrt{\frac{\gamma g_c}{RT}} = ¢ \qquad (*2.4,6)$$

c. Differentiating the "Energy" equation, Equation (1.19) in the text, and neglecting the potential term, we can show for steady flow that:

$$d[h + \frac{V^2}{2g_c}] = \delta q - \delta w_s \qquad (*2.4,8)$$

You saw that for a perfect gas:

$$a^2 = \gamma g_c RT \qquad (*2.4,9)$$

hence

$$d[h + \frac{M^2(\gamma RT)}{2}] = \delta q - \delta w_s \qquad (*2.4,10)$$

We can re-write this expression as:

$$d[h + (\frac{\gamma-1}{2}) \frac{M^2(\gamma RT)}{(\gamma-1)}] = \delta q - \delta w_s \qquad (*2.4,11)$$

From section 1.11 recall that for a perfect gas:

$$(\frac{\gamma}{\gamma-1}) = \frac{c_p}{R} \qquad (*2.4,12)$$

and

$$dh = c_p dT \qquad (*2.4,13)$$

Show that:

$$d[h(1 + \frac{\gamma-1}{2} M^2)] = \delta q - \delta w_s \qquad (*2.4,14)$$

Now use the product rule for differentials to show that:

$$(1 + \frac{\gamma-1}{2} M^2)\, dh + \frac{\gamma-1}{2} h dM^2 = \delta q - \delta w_s \qquad (*2.4,15)$$

d. We derived in Unit 1.6 of the Study Guide an expression for the "Momentum" equation in steady, one dimensional flow:

$$\frac{dP}{\rho} + \frac{fV^2}{D_E 2g_c} dx + \frac{gdz}{g_c} + \frac{VdV}{g_c} = 0 \qquad (*2.4,16)$$

Divide through by P, multiply each term by $\rho$, neglect the dz term, and rewrite the last term to yield:

$$\frac{dP}{P} + \frac{\rho f V^2}{PD_E 2g_c} + \frac{\rho dV^2}{P2g_c} = 0 \qquad (*2.4,17)$$

Now, substitute $\rho RT$ for P, multiply the second and third terms by $\gamma/\gamma$ and regroup the second term to obtain:

$$\frac{dP}{P} + \frac{\gamma}{2} M^2 \frac{f}{D_E} dx + \frac{\gamma dV^2}{2\gamma g_c RT} = 0 \qquad (*2.4,18)$$

The third term can be modified by considering the product rules for the differential of the two variables M and T:

$$\frac{\gamma}{2} \frac{dV^2}{a^2} = \frac{\gamma}{2} \frac{d(a^2M^2)}{a^2} = \frac{\gamma}{2} dM^2 + \frac{\gamma}{2} M^2 \frac{dT}{T} \qquad (*2.4,19)$$

So now the "Momentum" equation may be written:

$$\frac{dP}{P} + \frac{\gamma}{2} dM^2 + \frac{\gamma}{2} M^2 \frac{dT}{T} + \frac{\gamma}{2} M^2 (f \frac{dx}{D_E}) = 0 \qquad (*2.4,20)$$

## UNIT 3 - ISENTROPIC FLOW - OBJECTIVES

The student shall be able to:

1. Define an ISENTROPIC PROCESS and explain the relationship among reversible, adiabatic and isentropic processes.

2. Show graphically how pressure, density and area vary in steady, one-dimensional, isentropic flow as Mach Number ranges from zero to supersonic values.

3. Explain the difference between STATIC and STAGNATION properties. Given all the static properties, compute the stagnation properties.

4. Describe what is meant by a "choked" flow passage.

5. Compare the function of a NOZZLE and a DIFFUSER. Sketch physical devices that perform as each for subsonic and supersonic flow.

6. Simplify the basic equations for CONTINUITY, ENERGY and MOMENTUM to relate differential changes in density, pressure and volume to a differential change in area for steady, one-dimensional flow through a varying area passage.

7. Define the reference condition, (*) and the properties associated with it. (i.e., $A^*$, $P^*$, $T^*$, $\rho^*$, etc.)

8. Define stagnation temperature ($T_t$) of a perfect gas in terms of temperature (T), Mach Number (M) and ratio of specific heats ($\gamma$).

9. Define stagnation pressure ($P_t$) of a perfect gas in terms of temperature (T), Mach Number (M) and ratio of specific heats ($\gamma$).

10. Define stagnation enthalpy ($h_t$) in terms of enthalpy (h) and velocity (V). Put in terms of M and $\gamma$ for a perfect gas.

11. Depict graphically the general relationships of velocity, density and area in varying area adiabatic flow where the Mach Number varies from zero to supersonic values.

12. Express the loss relationship in adiabatic flow ($\Delta s_i$) as a function of stagnation pressures ($P_t$) or reference areas ($A^*$) between two points in the flow.

13. Cite the necessary conditions for the reference area $A^*$ to remain constant for all points in the flow field.

14. Derive the working equations for a perfect gas relating property ratios between two points, in both adiabatic and isentropic flows, as a function of Mach Number (M), ratio of specific heats ($\gamma$) and change in entropy ($\Delta s$).

15. State and interpret the relation between stagnation pressure ($P_t$) and the reference area ($A^*$) for the process between two points in adiabatic flow.

16. Utilize the adiabatic and isentropic flow relations and the isentropic tables to solve typical flow problems.

STUDY GUIDE - UNIT 3

## ADIABATIC AND ISENTROPIC FLOW

> Engineering problems typically involve many factors which affect the fluid flow conditions and frequently one of these factors is predominant. In this case it is possible to consider only the controlling factor and develop a simple solution to the problem. These approximate solutions can be applied with great confidence to many engineering situations. To the engineering student, this approach also provides insight concerning the significance, magnitude and relative importance of each of the factors considered, as the basic state variables are changed.

### 3.1 - Variable Area Flow in General

a. We will now consider flow in a variable area channel without heat transfer, or "adiabatic" flow. Many real life situations fit into this category. We have learned that for many situations air and other gases flowing in a continuous field obey the Perfect Gas Law reasonably well. Since this is the case, we will develop working equations using the 'perfect gas' assumption. Additionally, we are motivated, for the sake of simplicity, to continue to assume steady, one-dimensional flow with no work (that is, shaft work) done on or by the system.

   If we consider the adiabatic flow as it occurs in real situations, some accounting must be made for losses which occur as a result of friction and other irreversible factors. When these factors are considered negligible and the flow is taken to be reversible, the flow meets the criteria for isentropic flow. This concept was covered in brief in the Review Questions. We may view the isentropic flow case as the 'ideal' or 'Standard' with which to measure all adiabatic flow.

b. Read sections 3.1 and 3.2 in the text.

c. What common simplification is used to obtain equations 3.1 and 3.2 from the basic equations? When is it justifiable to neglect the higher ordered terms and assume that these versions of the basic equations (3.1, 3.2, etc.) are correct?

d. To gain some insight into the manner in which the flow variables are changing when a fluid encounters a variable area, it will be convenient to restrict the analysis to flow with no losses. We shall start with the energy equation:

$$\delta q = \delta w_s + dh_t \qquad (*3.1,1)$$

Since we have made the assumptions of no heat transfer or shaft work:

$$dh_t = 0 \qquad (*3.1,2)$$

Recalling the definition of stagnation enthalpy (neglecting potential energy change):

$$h_t = h + \frac{V^2}{2g_c} \qquad (*3.1,3)$$

Differentiating:

$$dh_t = dh + \frac{VdV}{g_c}$$

but:

$$dh_t = 0$$

so:

$$dh = -\frac{VdV}{g_c} \qquad (*3.1,4)$$

Now, we may start with equation (1.22), from the Second Law of Thermodynamics:

$$Tds = dh - \frac{dP}{\rho} \qquad (*3.1,5)$$

Since this process is both reversible <u>and</u> adiabatic, it is <u>isentropic</u>. Thus ds = 0.

So finally:

$$dh = \frac{dP}{\rho} \qquad (*3.1,6)$$

Considering the equations (*3.1,4) and (*3.1,6) we obtain

$$-\frac{VdV}{g_c} = \frac{dP}{\rho}$$

or

$$dV = -\frac{g_c dP}{\rho V} \qquad (*3.1,7)$$

We introduce this into equation 3.1 and the differential form of the continuity equation becomes:

$$\frac{d\rho}{\rho} + \frac{dA}{A} - \frac{g}{\rho}\frac{dP}{V^2} = 0 \qquad (*3.1,8)$$

which can be solved for

$$\frac{dP}{\rho} = \frac{V^2}{g_c}\left[\frac{d\rho}{\rho} + \frac{dA}{A}\right] \qquad (*3.1,9)$$

The definition of sonic velocity:

$$a^2 = g_c \left.\frac{\partial P}{\partial \rho}\right|_{s=constant}$$

can be written for our case as:

$$a^2 = g_c \frac{dP}{d\rho} \qquad (2.3)$$

because this flow is isentropic.

Now, rearranging equation 2.3:

$$dP = \frac{a^2}{g_c} d\rho \qquad (*3.1,10)$$

Substituting this expression for dP into equation *3.1,9 yields:

$$\frac{d\rho}{\rho} = \frac{V^2}{a^2} \left[ \frac{d\rho}{\rho} + \frac{dA}{A} \right] \qquad (*3.1,11)$$

From the definition of Mach Number

$$M^2 = \frac{V^2}{a^2}$$

show that:

$$\frac{d\rho}{\rho} = \left[ \frac{M^2}{1-M^2} \right] \frac{dA}{A} \qquad (*3.1,12)$$

If we now substitute equation *3.1,12 into equation 3.1 in the the text you can show that:

$$\frac{dV}{V} = - \left[ \frac{1}{1-M^2} \right] \frac{dA}{A} \qquad (*3.1,13)$$

But we recall that:

$$dV = \frac{-g_c}{\rho V} dP \qquad (*3.1,7)$$

or

$$\frac{dV}{V} = - \frac{g_c}{\rho V^2} dP \qquad (*3.1,7a)$$

Substitute this expression for $\frac{dV}{V}$ into equation *3.1,13 and solve for dP:

$$dP = \frac{\rho V^2}{g_c} \left[ \frac{1}{1-M^2} \right] \frac{dA}{A} \qquad (3.5)$$

We now collect the principal relations which will be referred to in the following sections.

$$\boxed{dP = \frac{\rho V^2}{g_c} \left[ \frac{1}{1-M^2} \right] \frac{dA}{A}} \qquad (3.5)$$

$$\boxed{\frac{d\rho}{\rho} = \left[ \frac{M^2}{1-M^2} \right] \frac{dA}{A}} \qquad (*3.1,12)$$

$$\boxed{\frac{dV}{V} = - \left[ \frac{1}{1-M^2} \right] \frac{dA}{A}} \qquad (*3.1,13)$$

e. Consider now what must be happening when fluid flows through the variable area channel. Fill in the blanks below. Assume that the pressure is always decreasing. Thus dP is negative.

From equation 3.5 you see that if $M < 1$, dA must be ________ indicating that the area is ___________, whereas if $M > 1$, dA must be ___________ and the area is ___________.

From here you can move to equation *3.1,12. If $M < 1$ and dA _________ then $d\rho$ must be _________. If $M > 1$ and dA _________ then $d\rho$ must be _________.

Looking at equation *3.1,13 reveals that if $M < 1$ and dA ________ then dV must be ________ meaning that velocity is __________, whereas if $M > 1$ and dA ________ then dV must be __________ and velocity is ___________.

f. We can summarize the effects and variation of velocity, density and area as Mach Number varies from zero to supersonic values. Refer to Figure *3.1 and recall the continuity equation:

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0 \qquad (*1.41)$$

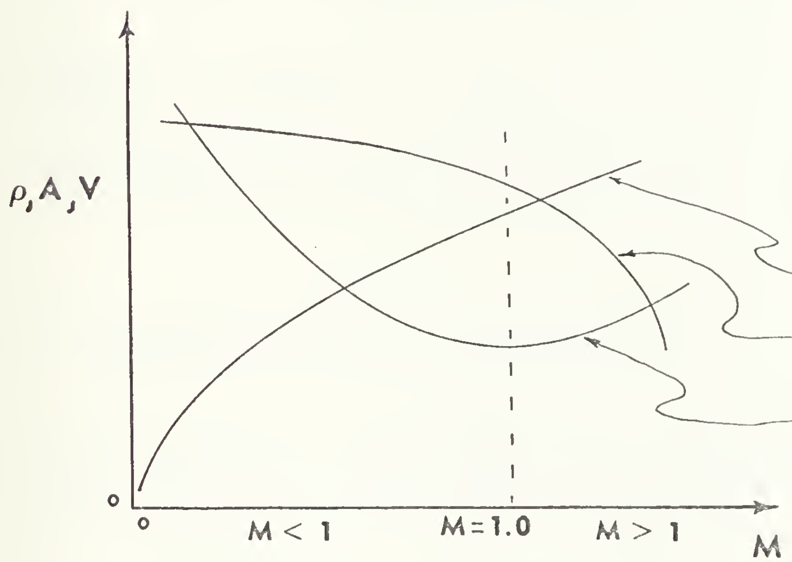



Figure *3.1

Summary of variation of ρ, V and A.

| M << 1 | M = 1 | M > 1 |
|---|---|---|
| 1. Density ≃ constant<br><br>2. Area compensates Velocity | 1. Density compensates Velocity<br><br>2. dA = 0 | 1. Density compensates Area and Velocity |

g. Now read sections 3.3 and 3.4 in the text. Consider carefully the results pictured in Figures 3.3, 3.4 and 3.5. The situation depicted in Figure 3.5 in the text can only exist if the proper pressure conditions exist <u>throughout</u> the nozzle.

h. A 'nozzle' is a device which converts enthalpy (or pressure energy, for the case of incompressible fluids) into kinetic energy. A 'diffuser' is a device which converts kinetic energy into enthalpy. This distinction is often overlooked and a device is labeled a "nozzle" simply because it is a convergent section, with no regard for the flow conditions.

## 3.2 - Computation of Stagnation Conditions for a Perfect Gas

a. Read the first part of Section 3.4 (from page 39 to the middle of page 41).

b. Particularly note equations 3.6 and 3.7. These will be used frequently and should be learned.

## 3.3 - Adiabatic Flow for a Perfect Gas

a. We now return to the problem of adiabatic varying area flow and include any losses that might exist. To make this problem tractable we will assume that the fluid is a perfect gas.

b. From earlier work recall the following relations which hold true for a perfect gas:

$$\Delta h = c_p \Delta T \qquad (1.25)$$

$$c_p - c_v = R \qquad (1.26)$$

$$\left.\begin{aligned} \frac{c_p}{c_v} &= \gamma \\ c_p &= \frac{\gamma}{\gamma-1} R \end{aligned}\right\} \text{From section 1.11 in text}$$

From 3.1 we showed that

$$\delta q = \delta w_s + dh_t \qquad (*3.1,1)$$

So for adiabatic flow with no work:

$$dh_t = 0 \qquad (*3.1,2)$$

or

$$h_t = ¢$$

This is true for any fluid under conditions where $Q = W_s = 0$. From equation 1.25, if one considers a perfect gas, the stagnation temperature ($T_t$) is also constant.

c. Review the relations we already know to determine which ones may be useful in analyzing adiabatic flow. First consider the continuity equation for steady, one-dimensional flow:

$$\dot{m} = \rho AV = ¢ \qquad (*3.3,1)$$

Examine the flow between two arbitrary points in a flow system such as in Figure *3.2.

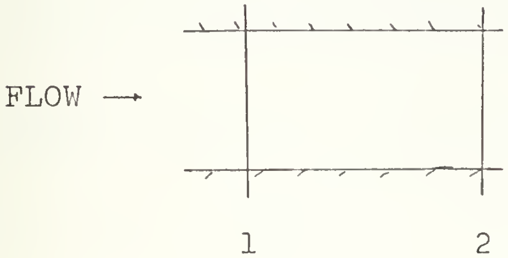



Figure *3.2

Equation *3.3,1 may be written:

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \qquad (*3.3,2)$$

or

$$\frac{A_2}{A_1} = \frac{\rho_1 V_1}{\rho_2 V_2} \qquad (*3.3,3)$$

From the definition of Mach Number:

$$V = Ma \qquad (*3.3,4)$$

and the expression for the speed of sound in a perfect gas:

$$a = \sqrt{\gamma g_c RT} \qquad (*3.3,5)$$

and recalling the equation of state for a perfect gas:

$$P = \rho RT \qquad (*3.3,6)$$

we can re-write equation *3.3,3 as:

$$\frac{A_2}{A_1} = \frac{P_1 M_1}{P_2 M_2} \left[ \frac{T_2}{T_1} \right]^{1/2} \qquad (*3.3,7)$$

Work out the intermediate steps from *3.3,3 to *3.3,7 in order to corroborate this expression.

d. We will now turn to the energy equation. We have seen that:

$$h_t = h + \frac{V^2}{2g_c} \qquad (*3.3,8)$$

Hence the energy equation may be written between two points as:

$$h_{t_1} + q_{1\to2} = w_{s_{1\to2}} + h_{t_2} \qquad (*3.3,9)$$

Since $T_{t_1} = T_{t_2}$ in a perfect gas under adiabatic flow conditions with no shaft work, you can now re-write equation 1.25 from the text, between any arbitrary point in the system and stagnation conditions. You should get:

$$h_t - h = c_p [T_t - T] \qquad (*3.3,10)$$

Now using equation *3.3,8 we can show that

$$T_t = T [1 + \frac{\gamma-1}{2} M^2] \qquad (3.6)$$

The development of this equation is found in section 3.4 in the text.

Since $T_t$ is a constant between any two points in the flow, we can now write:

$$T_1 [1 + \frac{\gamma-1}{2} M_1{}^2] = T_2 [1 + \frac{\gamma-1}{2} M_2{}^2] \qquad (*3.3,11)$$

or

$$\frac{T_2}{T_1} = \frac{1 + \frac{\gamma-1}{2} M_1{}^2}{1 + \frac{\gamma-1}{2} M_2{}^2} \qquad (*3.3,12)$$

e. From the stagnation energy equation for adiabatic flow of a perfect gas with no work :

$$\frac{dP_t}{P_t} + \frac{ds_i}{R} = 0 \qquad (*3.3,13)$$

If we integrate both sides between two points (1 & 2), we get:

$$\ln \frac{P_{t2}}{P_{t1}} = \frac{-(s_{i_2} - s_{i_1}}{R} \qquad (*3.3,14)$$

Since $ds_e = 0$, $\Delta s_i = \Delta s$ so, taking the anti-log of both sides of equation *3.3,14 yields:

$$\boxed{\frac{P_{t2}}{P_{t1}} = e^{-\frac{\Delta s}{R}}} \qquad (*3.3,15)$$

This expression is of particular value.

f. Stagnation pressure is defined as the pressure obtained if the flow is brought to rest <u>isentropically</u>.

Between two points we can recall from section 1 (for an isentropic process):

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} \qquad (1.30)$$

Let condition 2 be the stagnation state and condition 1 be the static state. Use equations 1.30 and 3.6 to show that:

$$\frac{P_t}{P} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}} \qquad (3.7)$$

Thus between two points:

$$\frac{P_{t2}}{P_{t1}} = \frac{P_2\left(1 + \frac{\gamma-1}{2} M_2^{\,2}\right)^{\frac{\gamma}{\gamma-1}}}{P_1\left(1 + \frac{\gamma-1}{2} M_1^{\,2}\right)^{\frac{\gamma}{\gamma-1}}} = e^{-\frac{\Delta s}{R}} \qquad (*3.3,16)$$

Rearrange this equation to show that:

$$\frac{P_1}{P_2} = \left[\frac{1 + \frac{\gamma-1}{2} M_2^{\,2}}{1 + \frac{\gamma-1}{2} M_1^{\,2}}\right]^{\frac{\gamma}{\gamma-1}} \left(e^{+\frac{\Delta s}{R}}\right) \qquad (*3.3,17)$$

We can now utilize equation *3.3,17 to obtain a relation which expresses the change in area in terms of Mach Number and a loss term.

From continuity show that:

$$\frac{A_2}{A_1} = \frac{P_1 M_1}{P_2 M_2}\left[\frac{T_2}{T_1}\right]^{1/2} \qquad (*3.3,18)$$

You can now substitute from equations *3.3,17 and *3.3,12 to obtain:

$$\frac{A_2}{A_1} = \left\{ \left[ \frac{1 + \frac{\gamma-1}{2} M_2{}^2}{1 + \frac{\gamma-1}{2} M_1{}^2} \right]^{\frac{\gamma}{\gamma-1}} \left( e^{\frac{\Delta s}{R}} \right) \right\} \frac{M_1}{M_2} \left[ \frac{1 + \frac{\gamma-1}{2} M_1{}^2}{1 + \frac{\gamma-1}{2} M_2{}^2} \right]^{1/2}$$

(*3.3,19)

By combining terms, equation *3.3,19 can be written:

$$\frac{A_2}{A_1} = \frac{M_1}{M_2} \left[ \frac{1 + \frac{\gamma-1}{2} M_2{}^2}{1 + \frac{\gamma-1}{2} M_1{}^2} \right]^{\frac{\gamma+1}{2(\gamma-1)}} \left( e^{\frac{\Delta s}{R}} \right) \qquad (*3.3,20)$$

## 3.4 - Isentropic Tables

a. Read the remainder of section 3.4 in the text. Note the introduction of a new reference condition denoted by *. At this location the Mach Number is unity and this condition can be reached by many processes. This is a hypothetical reference condition which may or may not exist in the system (as was the case for the stagnation condition).

b. One can now simplify many general equations that have been developed by assuming no losses.

Consider an isentropic flow case viewed in the T-- S plane.

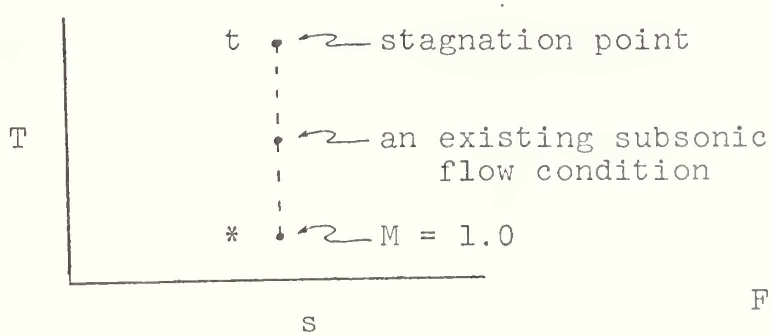



Figure *3.3

By utilizing the expressions for area, pressure and temperature ratios and letting one state point be a reference condition, if we know what the fluid is (i.e., the value of $\gamma$) then we can construct tables which relate these ratios to various values of Mach Number for the <u>isentropic</u> case.

As an example, let 2 be an arbitrary point in the flow system and let 1 be the stagnation reference state. Then

$T_2 = T$ (hence $M_2 = M$ = any value)

$T_1 = T_t$ (hence $M_1 = 0$)

and equation *3.3,12 can be written as:

$$\frac{T}{T_t} = \frac{1}{1 + \frac{\gamma-1}{2} M^2} = f(M,\gamma) \qquad (*3.4,1)$$

As another example, let 2 be an arbitrary point in the flow system and let 1 be the reference state *.

Then: $A_2 = A$ (hence $M_2 = M$ = any value)

$A_1 = A^*$ (hence $M_1 = 1$)

Remembering that we are considering an isentropic process, equation *3.3,20 can be written as:

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{1 + \frac{\gamma-1}{2} M^2}{\frac{\gamma+1}{2}}\right]^{\frac{\gamma+1}{2(\gamma-1)}} = f(M,\gamma) \qquad (*3.4,2)$$

c. We see that for isentropic processes these important ratios are simply functions of Mach Number and gamma ($\gamma$) which lead to tabulation of them in Appendix A.

d. Evaluate equation *3.4,1 for an arbitrary value of $M > 1$ and an arbitrary value of $M < 1$. Compare your answers with the values for $T/T_t$ in the appendix. Notice that the tables are dimensionless and are always expressed in terms of a ratio of pressures, temperatures, etc.

3.5 - Relations Between $P_t$ and $A^*$

a. We can now relate the stagnation pressure ($P_t$) and the sonic reference area ($A^*$). Take $A_1$ as $A_1^*$ and take $A_2$ as $A_2^*$ in equation *3.3,20. By definition $M_1 = M_2 = 1$. This expression reduces to

$$\frac{A_2^*}{A_1^*} = e^{\frac{\Delta S}{R}} \qquad (*3.5,1)$$

We saw that:

$$\frac{P_{t2}}{P_{t1}} = e^{-\frac{\Delta S}{R}} \qquad (*3.3,15)$$

Thus

$$\left(\frac{P_{t2}}{P_{t1}}\right)\left(\frac{A_2^*}{A_1^*}\right) = \left(e^{\frac{\Delta S}{R}}\right)\left(e^{-\frac{\Delta S}{R}}\right) = 1 \qquad (*3.5,2)$$

or

$$\boxed{P_{t2}\,A_2^* = P_{t1}A_1^* = \text{constant}} \qquad (*3.5,3)$$

Note that equations *3.5,2 and *3.5,3 apply to any adiabatic situation of flow of a perfect gas where $w_s = 0$.

b. It should be noted (and we can verify from equation *3.3,20) that for two different points in the flow field (1 & 2), $A_1^*$ is only equal to $A_2^*$ if the flow is isentropic between points 1 & 2.

c. Now read sections 3.5, 3.6 and 3.7 in the text.

## UNIT 4 - NORMAL and OBLIQUE SHOCKS - Objectives

The student shall be able to:

1. List the assumptions used to analyze a standing normal shock.

2. Sketch a "shock process" on a T-S diagram, noting the pertinent features of the phenomenom.

3. Explain why an "expansion shock wave" cannot exist.

4. Recognize the necessary and sufficient conditions to solve "normal shock" problems.

5. Derive the expression which describes the properties on the downstream side of a standing normal shock in terms of properties on the upstream side.

6. Explain how shock tables may be developed from the relations describing the properties on the downstream side of a standing normal shock in terms of the properties on the upstream side.

7. Demonstrate the ability to solve typical normal shock problems by use of the tables and/or normal shock equations.

8. Explain how an oblique shock can be described by the superposition of a normal shock and another flow field.

9. Identify which properties remain constant and which change when a uniform velocity is superimposed on the flow field.

10. Show (by diagrams) how the "shock angle" and the "deflection angle" are defined.

11. Describe the general results of an oblique shock analysis in terms of a diagram such as "shock angle" versus "Mach number" for various deflection angles.

12. Distinguish between weak and strong shocks. Know what conditions cause each to form.

13. Describe the conditions which cause a detached shock to form.

14. Solve typical problems involving oblique shocks (such as air inlets, nozzle outlets, wedges, etc.)

15. Describe the treatment of a moving normal shock wave so as to apply the relations developed for the standing normal shock.

## Unit 4 - NORMAL and OBLIQUE SHOCKS

### 4.1 - The Shock Mechanism

a. To this point we have treated fluid flow problems which occured in a continuous medium; that is, wherein no discontinuities existed within the control volume under consideration. We now turn to consideration of a <u>discontinuous</u> process - the "shock". A shock wave characteristically occurs within a very thin, but finite volume. The thickness of a shock is on the order of $10^{-5}$ inches. Due to the complex interactions involved, analysis of the properties within the shock are beyond the scope of this course. We will analyze the change in fluid properties across the shock by investigating the behavior on both sides of a shock wave.

b. Now read Sections 4.1 and 4.2 in the text.

c. We will first consider a "standing (i.e. NON-MOVING) normal shock"; that is, a standing shock which is perpendicular to flow. With the principles developed in this analysis we will then investigate the possibility of the existence of an expansion shock. Then moving normal shocks, convergent-divergent nozzle operation, and oblique shocks will be covered.

## 4.2 - Standing Normal Shock

a. Consider a flow situation as shown in Figure *4.1. As with previous analyses we first establish a control volume,

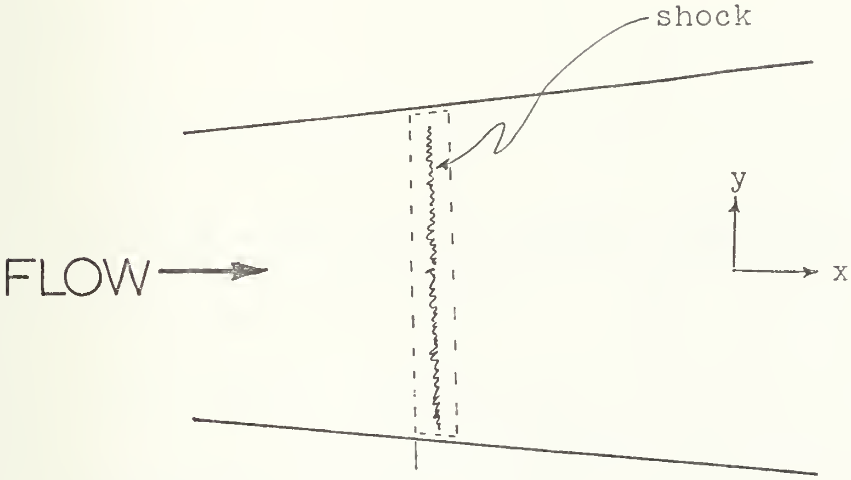



Figure *4.1

then apply the basic equations to the properties across the control volume.

b. The following assumptions will be made:

1) Since the shock is very, very thin, we may consider the control volume equally thin and assume that there is no surface along the wall to create friction.

2) Due to the thin control volume, the cross-sectional area on both sides of the shock may be considered to be essentially equal.

3) We will continue to assume steady, one-dimensional flow with negligible potential changes.

c. Consider first the continuity equation. Since this is steady, one-dimensional flow:

$$\dot{m} = \text{constant} \qquad (*4.2,1)$$

or

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

but

$$A_1 = A_2$$

so

$$\rho_1 V_1 = \underset{?}{\underline{\qquad\qquad}} \qquad (*4.2,2)$$

d. Now recall the energy equation for steady, one-dimensional flow between two points:

$$h_{t_1} + q_{1\to 2} = h_{t_2} + w_{s_{1\to 2}} \qquad (*2.1,7)$$

We can consider this an adiabatic process with no shaft work being done, so $q_{1\to 2} = w_{s_{1\to 2}} = 0$, or

$$h_{t_1} = \underset{?}{\underline{\qquad\qquad\qquad}} \qquad (*4.2,3)$$

e. The x-component of the momentum equation for <u>steady, one-dimensional</u> flow (Equation 1.13) becomes:

$$\sum F_x = \int_{c.v.} \frac{\rho}{g_c} V_x (\vec{V}.\vec{dA}) \qquad (*4.2,4)$$

Integrating the right side of equation *4.2,2 yields

$$\int \frac{\rho}{g_c} V_x (\vec{V}.\vec{dA}) = \frac{\dot{m}}{g_c} [V_2 - V_1]$$

We can also express the summation of forces in the x-direction as

$$\sum F_x = P_1 A_1 - P_2 A_2 = (P_1 - P_2)A$$

So equation *4.2,4 becomes

$$(P_1 - P_2)A = \frac{\dot{m}}{g_c} [V_2 - V_1] \qquad (*4.2,5)$$

but $\dot{m} = \rho AV$ so show that:

$$P_1 + \frac{\rho_1 V_1^{\,2}}{g_c} = P_2 + \frac{\rho_2 V_2^{\,2}}{g_c} \qquad (*4.2,6)$$

f. To this point we have been dealing with a general fluid. By introducing the Perfect Gas Law

$$P = \rho RT \qquad (1.23)$$

and recalling the expression for the speed of sound in a Perfect Gas (Equation *3.3,4):

$$V = Ma = M\sqrt{\gamma g_c RT} \qquad (*4.2,7)$$

We can now express Equation *4.2,2 in terms of Mach number

$$\frac{P_1 M_1}{\sqrt{T_1}} = \frac{P_2 M_2}{\sqrt{T_2}} \qquad (*4.2,8)$$

Obtain this expression yourself for drill.

Note: $\gamma$ is assumed constant for this expression.

Similarly, recalling from Equation (3.6)

$$T_t = T[1 + \frac{\gamma-1}{2} M^2] \qquad (3.6)$$

and since

$$h_t = c_p T_t$$

from Section 1.1 in the text, the energy equation (Equation *4.2,3) can be written

$$T_1[1 + \frac{\gamma-1}{2} M_1^2] = T_2[1 + \frac{\gamma-1}{2} M_2^2] \qquad (*4.2,9)$$

g. Similarly, using the definition of Mach Number, the Perfect Gas Law and the expression for speed of sound in a perfect gas, show that the Momentum Equation (Equation (4.2,6) can be written:

$$P_1[1 + \gamma M_1^2] = P_2[1 + \gamma M_2^2] \qquad (*4.2,10)$$

h. We have in effect seven variables:

$\gamma$, $P_1$, $M_1$, $T_1$, $P_2$, $M_2$ and $T_2$

and three governing equations, nos. *4.2,8, *4.2,9, and *4.2,10. Given any 4 of these we should be able to solve for the remaining three variables.

i. We now find it possible to combine the three basic equations so as to derive an expression for $M_2$ in terms of $M_1$ and $\gamma$. Rearrange the continuity and energy equations as follows:

$$\frac{P_1 M_1}{P_2 M_2} = \sqrt{\frac{T_1}{T_2}} \qquad (*4.2,8a)$$

$$\sqrt{\frac{T_1}{T_2}} = \left[\frac{1 + \frac{\gamma-1}{2} M_2^2}{1 + \frac{\gamma-1}{2} M_1^2}\right]^{1/2} \qquad (*4.2,9a)$$

and the momentum equation:

$$\frac{P_1}{P_2} = \frac{1 + \gamma M_2^2}{1 + \gamma M_1^2} \qquad (*4.2,10a)$$

Now combine these three and show that:

$$\left[\frac{1+\gamma M_2^{\,2}}{1+\gamma M_1^{\,2}}\right]\frac{M_1}{M_2} = \left[\frac{1+\frac{\gamma-1}{2}M_2^{\,2}}{1+\frac{\gamma-1}{2}M_1^{\,2}}\right]^{1/2} \qquad (*4.2,11)$$

Solve for $M_2$ and obtain:

$$M_2^{\,2} = \frac{M_1^{\,2}+\frac{2}{\gamma-1}}{\left(\frac{2}{\gamma-1}\right)M_1^{\,2}-1} \qquad (*4.2,11a)$$

So for a flow situation such as in Figure *4.1, knowing the conditions at (1) (i.e., before the shock) and solving for $M_2$ we can utilize equations *4.2,10a; *4.2,7; *4.2,6; *3.3,15; *4.2,9a; and equations 3.6 and 3.7 from the text to solve for all other properties.

> REPEAT! If the properties are known ahead of the shock (condition 1) then ALL conditions after the shock (condition 2) can easily be found.

For a given fluid (i.e., $\gamma$ is known) since the property ratios are functions of $\gamma$ and $M_1$ only, it is possible to pre-compute the ratios of the fluid properties, and tabularize them. Appendix B in the text is such a table. Take $M_1$ to be 2.0 and solve for $M_2$, and the property ratios, using the relations we have developed. Compare your answers with the values in the Normal Shock table.

j. Notice that it is possible to use a subsonic value of $M_1$ and obtain a MATHEMATICALLY correct answer for $M_2$ which is INVALID. Why?

> HINT: Take $M_1$ to be .577 and solve for $\frac{P_{t2}}{P_{t1}}$
>
> Now what sort of value must $\Delta s$ (in this case $\Delta s_i$) have to satisfy Equation *3.3,15? This proves that an "expansion shock" is NOT POSSIBLE.

k. Examine Appendix B in the text. Note that as $M_1$ increases $M_2$ decreases and

$$\frac{P_2}{P_1}, \quad \frac{\rho_2}{\rho_1} \quad \text{and} \quad \frac{T_2}{T_1} \quad \text{are all} > 1.0$$

Therefore a shock is always a compression process, with $P_2$ always greater than $P_1$. Notice that $P_{t2}$ is always <u>less than</u> $P_{t1}$ (though for values of $M_1$ near 1.0 only slightly).

Thus a shock process is <u>IRREVERSIBLE</u>.

l. Read Section 4.3 in the text and study the examples.

## 4.3 - Moving Normal Shocks

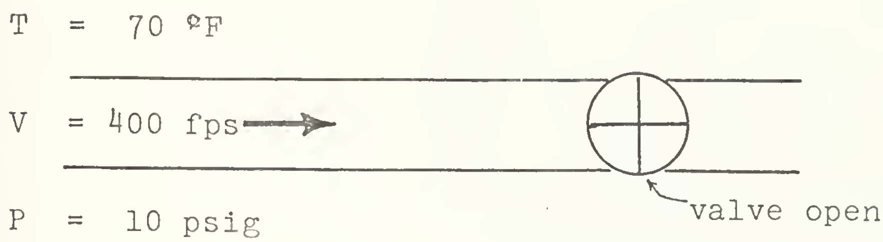



Figure *4.2

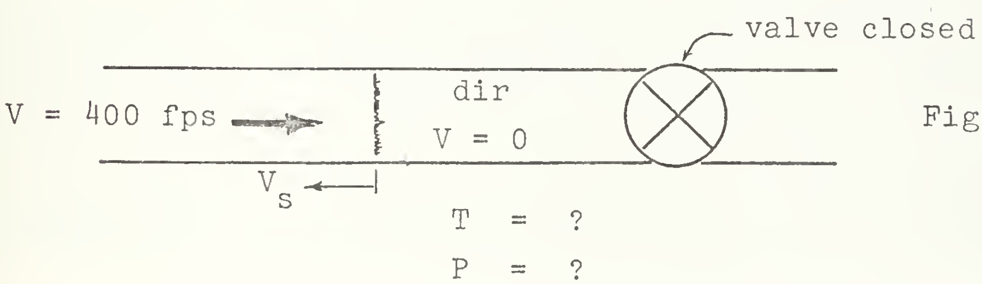
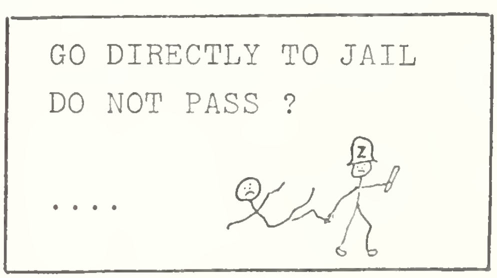



Figure *4.2a

a. Now consider a situation as pictured in Figure *4.2 where air is flowing in a pipe at the conditions shown and the valve at the end is <u>suddenly</u> closed. This will cause a shock wave to propagate back through the duct as pictured in *4.2a. We do <u>not</u> know the speed of this shock wave. Is this a steady flow problem? If you said "Yes",




and while you're there, re-read your handout entitled <u>Comments on Steady Flow</u>.

b. We can change this problem into a steady flow problem by "hopping aboard the shock wave."

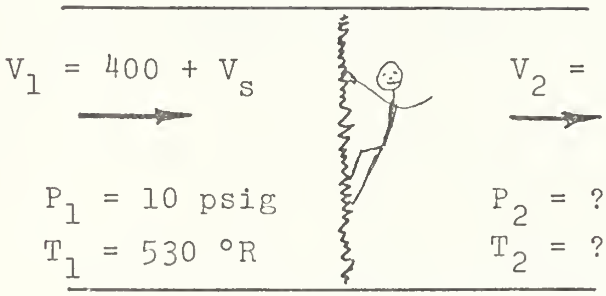



Figure *4.2b

We now have a steady flow standing normal shock problem. The velocity upstream of the shock wave will be ___________ and the speed downstream will be ___________. Why can we not derive a so-called closed form solution for $M_2$?

c. A "trial and error" process is required to solve this problem. Let us assume that $M_1 = 1.3$

$$a_1 = \sqrt{(1.4)(32.2)(53.3)(530)} = 1128 \text{ ft/sec}$$

Thus $V_1' = M_1 a_1 = (1.3)(1128) = 1467$ ft/sec

and $Vs = V_1 - 400 = \underline{1067}$ ft/sec

From the shock tables obtain $M_2 = .786$ and $\frac{T_2}{T_1} = 1.191$

Thus $T_2 = 1.191\ (530) = 632$ °R

$a_2 = 1233$ ft/sec

and $V_2 = M_2 a_2 = \underline{970}$ ft/sec

But $V_2$ must equal $V_s$, which it does not! Thus, our assumed Mach Number was in error. A few trials will converge on an assumption of $M_1 = 1.235$. Follow these calculations through and show that $V_2 = V_s \approx 992$ ft/sec.

What would the pressure be behind the shock wave? How do these values compare with those in the actual "unsteady" problem posed in Figure *4.2a?

d. Read Sections 4.4, 4.5 and 4.6 in the text.

## 4.4 - Convergent-Divergent Nozzle Operation

a. We recall that a convergent section acting as a nozzle will become "choked" at M = 1.

If the pressure downstream of such a nozzle is decreased the flow will accelerate until M = 1 is reached at the exit (see Figure *4.3).

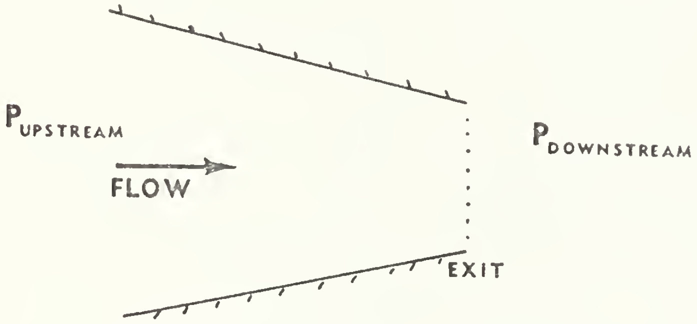



Figure *4.3

Further decrease in pressure outside the nozzle exit will not induce a further decrease in pressure inside the nozzle. The nozzle is said to be "choked".

We also recall (from Figure *3.1) that in order for a device to function as a 'nozzle' above M = 1.0 (i.e., continue to accelerate the flow, thus converting more enthalpy into kinetic energy) the cross-sectional area must begin to increase. Thus we can presage the existence of a convergent-divergent section (with M = 1.0 at the throat) which functions as a nozzle throughout its entire length. This is called a DeLaval nozzle.

De Laval Nozzle

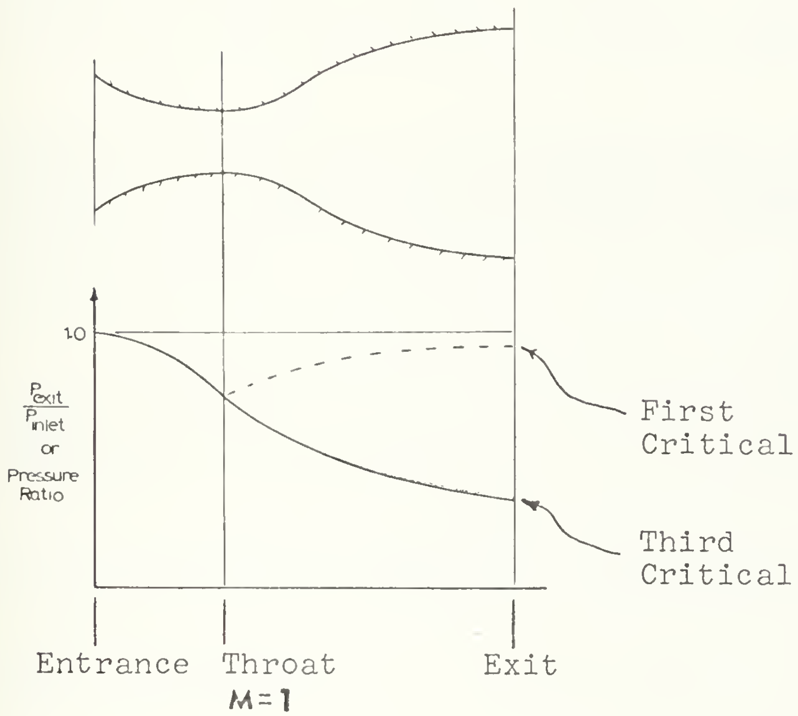



Figure *4.4

The convergent section of the C-D nozzle offers little design difficulty, but the divergent section must be carefully designed to reduce losses due to separation, wave interaction and other fairly complex phenomena.

b. Below the diagram of the C-D nozzle in Figure *4.4 is depicted a plot of static pressure ratio versus location in the nozzle. We can envision two possible flow conditions with M = 1 at the throat. Each of these conditions is associated with a pressure value at the exit known as a "critical point." The "First Critical Point" represents

flow which reaches $M = 1.0$ at the throat, but is subsonic in both the convergent and the divergent sections (dotted line). The "Third Critical Point" represents the design point (or pressure ratio) of the **C-D** nozzle. Both 1st and 3rd critical points represent isentropic flow conditions. Any exit pressure above 1st critical will result in subsonic flow throughout the nozzle, or typical "venturi" operation. A point between 1st and 3rd critical will induce non-isentropic flow through the nozzle.

c. Since both 1st and 3rd critical represent isentropic flow, Appendix A, "Isentropic Flow Tables", may be used to determine the values of Mach Number at first and third critical, provided we know the area ratio between the exit and the throat.

As an example suppose:

$$\frac{A_{exit}}{A_{throat}} = 3.0 \qquad (*4.4,1)$$

We know that at the throat $M = 1.0$ so $A_{throat} = A^*$, or:

$$\frac{A}{A^*} = 3.0 \qquad (*4.4,1a)$$

From Appendix A in the text:

for first critical $M_{exit} \approx .197$ $\quad \frac{P}{P_t} \approx .973$ (*4.4,2)

for third critical $M_{exit} \approx 2.64$ $\quad \frac{P}{P_t} \approx .0472$ (*4.4,3)

Two Notes on this solution
1) Linear interpolation can be used to obtain better accuracy.
2) The small figure (-1) indicates the power of ten with which the tabular value must be multiplied.

It is also interesting to compare these results to that of a converging (only) nozzle. In the case of a converging nozzle with $\gamma = 1.4$ we need a pressure ratio of .528.

($P_{exit}$ to $P_{inlet}$) to produce sonic velocity. In the above converging-diverging nozzle we have sonic velocity (in the throat) with a pressure ratio of only .97.

d. As you might expect, somewhere between "FIRST BASE" and "THIRD BASE" there must be a "SECOND BASE". Let's consider what happens in the region between "first critical" and "third critical."

e. As the receiver pressure is lowered below "first critical," a normal shock forms just downstream of the throat (see Figure *4.5). The remainder of the "nozzle" is now acting as a diffuser since the flow is subsonic and the area is increasing. The shock will locate itself in a position such that the pressure changes that occur ahead of the shock, across the shock, and downstream of the shock will produce a pressure that exactly matches the outlet pressure. In other words, the outlet pressure determines the location and strength of the shock. As the pressure is further lowered, the shock continues to move toward the exit. When the shock is located at the exit plane this condition is referred to as the 2nd critical point. If the receiver pressure is between 2nd and 3rd critical, then a <u>compression</u> takes place outside the nozzle. This is called "overexpansion" (i.e., the flow has expanded too far within the nozzle). If the receiver pressure is below 3rd critical then an <u>expansion</u> takes place outside the nozzle. This condition is called "underexpansion." We shall investigate these conditions later in the course.

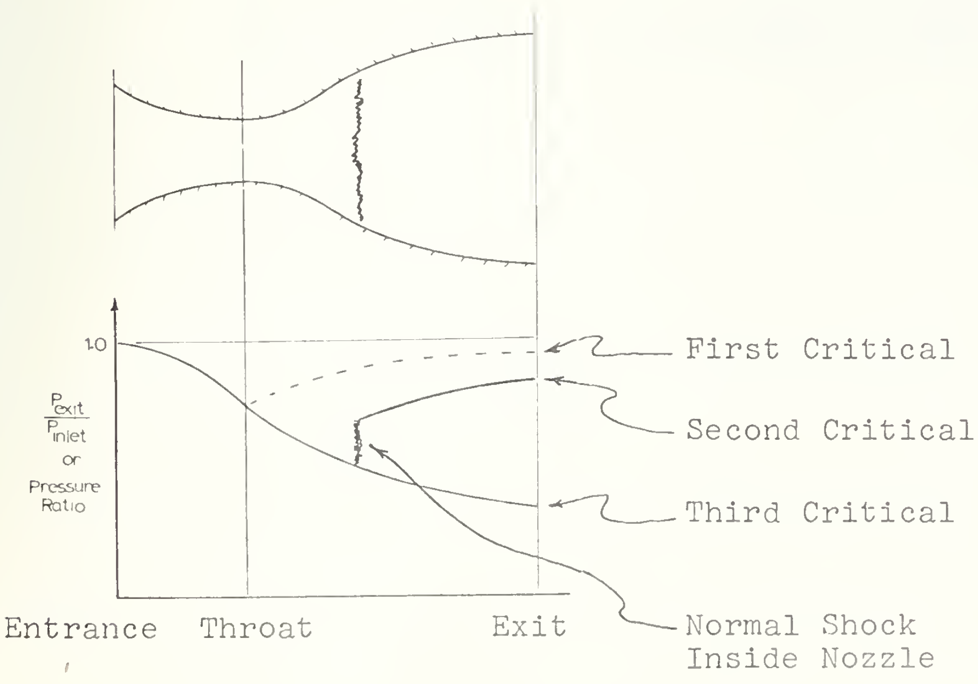



Figure *4.5

f. We can now use what we know about standing normal shocks to find the conditions at '2nd critical'. Just ahead ($\epsilon$ away", as the mathematicians are wont to say) of the shock at 2nd critical, the conditions are identical to those at 3rd critical (or design). If you have not done so, use the tables to confirm the exit Mach at 3rd critical is M = 2.64 for an Area ratio of

$$\frac{A_{exit}}{A_{throat}} = 3.0$$

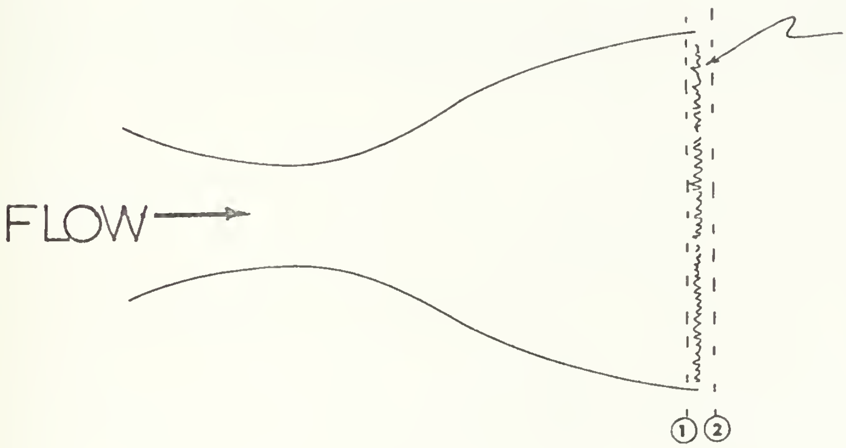



Figure *4.6

$$\frac{P_{exit}}{P_{inlet}} = \frac{P_2}{P_{t1}}$$

If we assume stagnation conditions at the inlet.

Is this a reasonable assumption?

From the Tables:

$$\frac{P_2}{P_{t2}} = \left(\frac{P_2}{P_1}\right)\left(\frac{P_1}{P_{t1}}\right) \qquad (*4.4,4)$$

From Isentropic Flow Tables @ M = 2.64

From Normal Shock Tables @ M = 2.64

Compute this and verify that:

$$\frac{P_{exit}}{P_{inlet}} = .376$$

g. Now suppose we subject the same De Laval nozzle to a pressure ratio of 0.4, whre will the shock form? Is this 'underexpanded' or 'overexpanded' Flow? Think about it before checking the solution below.

Solution

Since the flow is between 1st and 2nd critical, a normal shock has formed inside the nozzle — hence the terms "underexpanded" and "overexpanded" do not apply to the flow.

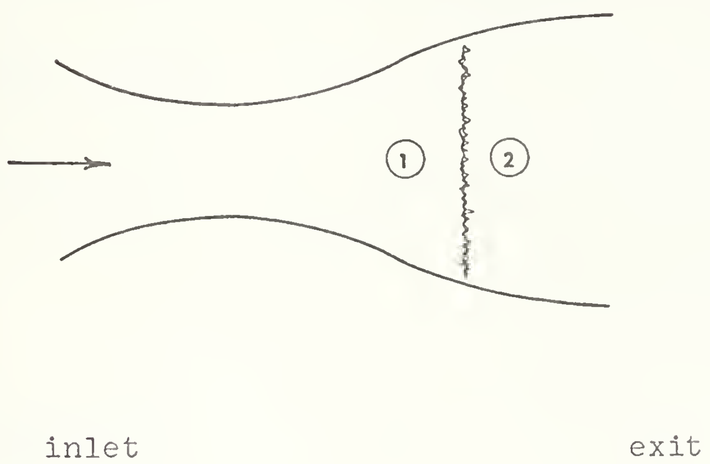



Figure *4.7

We have now enough information to analyze the flow, including losses.

Known

$$\frac{A_{exit}}{A_{throat}} = 3.0$$

$$\frac{P_{exit}}{P_{inlet}} = 0.4$$

But: $P_{inlet} = P_{t_{inlet}}$

and $A_{throat} = A^*_{inlet}$ $\Big\}$ since we assume no losses between the inlet and the throat.

Now recall Equation *3.5,3. Therefore:

$$A^*_{inlet} \; P_{t_{inlet}} = A^*_{exit} \; P_{t_{exit}} = \text{Constant} \qquad (*4.4,4)$$

Thus:

$$\left[\frac{A_{exit}}{A_{throat}}\right]\left[\frac{P_{exit}}{P_{inlet}}\right] = \left[\frac{A_{exit}}{A^*_{inlet}}\right]\left[\frac{P_{exit}}{P_{t_{inlet}}}\right] = \left[\frac{A_{exit}}{A^*_{exit}}\right]\left[\frac{P_{exit}}{P_{t_{exit}}}\right]$$

known known

$$(3.0) \quad (0.4) = 1.2$$

or:

$$\left[\frac{A_{exit}}{A^*_{exit}}\right]\left[\frac{P_{exit}}{P_{t_{exit}}}\right] = 1.2$$

From the equations developed in Unit 3:

$$\frac{A}{A^*} = f(M,\gamma) \qquad (*3.4,2)$$

$$\frac{P}{P_t} = f(m,\gamma) \qquad (3.7)$$

You can obtain an expression for

$$\frac{A}{A^*} \frac{P}{P_t} = f(M,\gamma) \qquad (*4.4,5)$$

John does not tabulate this value but a sample table including this function was passed out in class. Excerpts from a similar table for $\gamma = 1.4$ show:

| $M$ | $\frac{A}{A^*} \frac{P}{P_t}$ |
|---|---|
| .47 | 1.205 |
| .48 | 1.178 |

Interpolation reveals the Mach No. at the exit to be about

$$M_{exit} \approx .472$$

If we can determine the stagnation pressure ratio

$P_{t_{exit}} / P_{t_{inlet}}$ we will be able to locate the shock.

$$\frac{P_{t_{exit}}}{P_{t_{inlet}}} = \left(\frac{P_{t_{exit}}}{P_{exit}}\right)\left(\frac{P_{exit}}{P_{t_{inlet}}}\right) = \left(\frac{1}{.859}\right)\left(.4\right)$$

Given

From Isentropic tables for M = .472

Since all losses occur across the shock we can say that

$$\frac{P_{t_{exit}}}{P_{t_{inlet}}} = \left.\frac{P_{t_2}}{P_{t_1}}\right) \text{across shock} \qquad (*4.4,6)$$

From the shock tables find $M_1 = 2.59$ and from the isentropic tables find

$$\frac{A_{shock}}{A_{throat}} = 2.869$$

We also have a measure of the loss from

$$\frac{P_{t_2}}{P_{t_1}} = e^{-\frac{\Delta s}{R}} \qquad (*3.3,15)$$

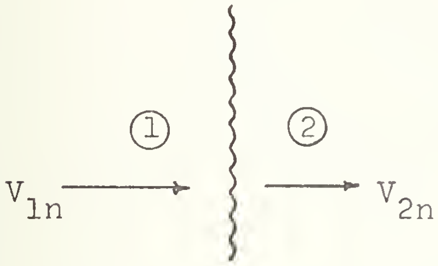
As we recall that $\Delta s = \Delta s_i$ here.

h. Read Section 5.2 and 5.3 in the text.

4.5 - Oblique Shocks

a. Read Section 6.1 in the text.

b. Consider a Standing Normal Shock



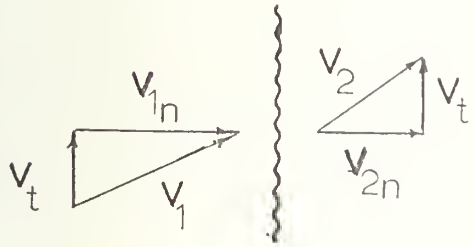

Recall: using Shock as Reference

$V_{1n}$ > Sonic

$V_{2n}$ < Sonic

Figure *4.8

Now, let us superimpose a velocity to $V_{1n}$ & $V_{2n}$ (say $V_t$) Note that this is the equivalent of running along the shock front. Recall that by doing this we have not changed the static states of the fluid, but we have changed the stagnation conditions. We then have:




Since $V_{2n} < V_{1n}$

and $V_t = V_t$

Notice that $V_2$ has turned towards the shock front.

Figure *4.9

We normally picture this as follows:

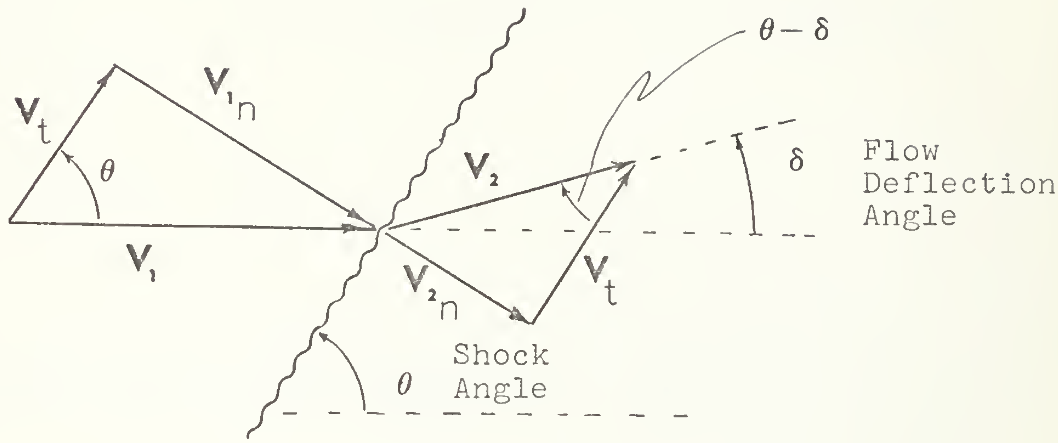



Figure *4.10

NOTE (1) Flow direction has changed
(2) $V_1 > V_2$ as with normal shock
$V_1$ must be supersonic (since $V_{1n}$ is supersonic
$V_2$ can now be supersonic – if $V_t$ is large enough

It should be clear that the Normal Shock Relations (and Tables) can be used IF, AND ONLY IF, proper care is taken.

Dividing both sides by $a_1$, and noting that $a_1 = a_{1n}$, we have:

$$\frac{V_{1n}}{a_{1n}} = \frac{V_1}{a_1} \sin \theta$$

Recalling the definition of Mach Number, this becomes

$$M_{1n} = M_1 \sin \theta$$

Static pressure ratios, static temperature ratios, etc. — can now be taken from the Normal Shock Tables, since these were unaltered by the superposition of $V_t$ on the original shock picture.

c. Now, we know that $M_{1n} \geq 1$

thus: $M_1 \sin \theta \geq 1$

Let us consider the range of possible $\theta$ for a given M: The minimum $\theta$ will occur when $M_1 \sin \theta = 1$

$$\theta_{min} = \sin^{-1} \frac{1}{M_1}$$

Recall that this is the same expression that was developed for the Mach Angle $\mu$. Hence, the Mach Angle is the minimum possible shock angle. Note that this is a limiting condition and really no shock exists for this case since then $M_{1n} = 1.0$. The maximum value that $\theta$ can achieve is obviously 90°. This is another limiting condition and represents our familiar normal shock.

Notice that as the shock angle $\theta$ decreases from 90° to the Mach angle $\mu$, $M_{1n}$ decreases from $M_1$ to 1. Since the strength of a shock is dependent upon the normal Mach number we have the means to produce a shock of any strength equal to or less than the normal shock.

c. Study Example 6.1 on pages 105-106 of the text.

d. We will now try to relate the deflection angle ($\delta$) to the shock angle ($\theta$) starting with the continuity equation.

$$\rho_1 V_{1n} = \rho_2 V_{2n} \qquad (*4.5,4)$$

so

$$\frac{\rho_1}{\rho_2} = \frac{V_{2n}}{V_{1n}} \qquad (*4.5,5)$$

and from equation (*4.5,1) and figure 6.5.

$$V_t = V_1 \cos\theta = V_2 \cos(\theta - \delta) \qquad (*4.5,6)$$

so

$$\frac{\rho_2}{\rho_1} = \frac{V_t}{\cos\theta}\left[\frac{\cos(\theta-\delta)}{V_t}\right]\frac{\sin\theta}{\sin(\theta-\delta)} = \frac{\tan\theta}{\tan(\theta-\delta)} \qquad (*4.5,7)$$

From our Normal Shock Relations in Unit 4.2, we can show that for a Perfect Gas (as a function of $M_{1n}$):

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)\,M_{1n}^{\;2}}{(\gamma-1)\,M_{1n}^{\;2} + 2} \qquad (*4.5,8)$$

but $M_{1n} = M_1 \sin\theta$ (*4.5,8a)

Combining equations *4.5,7, *4.5,8 and *4.5,8a yields:

$$\frac{(\gamma+1)\,M_1^{\;2}\sin^2\theta}{(\gamma-1)M_1^{\;2}\sin^2\theta + 2} = \frac{\tan\theta}{\tan(\theta-\delta)} \qquad (*4.5,9)$$

So $\theta = f(\delta, M, \gamma)$

or for a given $M_1$ and $\gamma$, the shock angle ($\theta$) is a function of the deflection angle ($\delta$). But note that this is a transcendental function. So for supersonic flow along a wall we cannot solve for $\theta$ as a function of $\delta$, M and $\gamma$ EXPLICITLY, but we can obtain an explicit solution for $\delta = f(\theta, M, \gamma)$.

$$\tan \delta = 2 \cot \theta \left[ \frac{M_1^2 \sin^2 \theta - 1}{M_1^2(\gamma + \cos 2\theta) + 2} \right] \qquad (*4.5,10)$$

Hence we can construct a chart such as in Appendix C to relate these three in accordance with Equation *4.5,10.

e. Now read the remainder of Section 6.2, and carefully study the examples.

f. Read Section 6.3 in the text.

g. Read Section 6.5. We summarize here some of the important concepts:

1. Flow always turns "toward" an oblique shock front.
2. For given values of $\delta$ and M, two values of $\theta$ may exist.
   a. A large pressure ratio results in a <u>strong</u> shock and <u>subsonic</u> $M_2$.
   b. A small pressure ratio results in a <u>weak</u> shock and supersonic $M_2$.
3. A maximum value of $\delta$ exists for a given Mach Number.
4. For $\delta > \delta_{max}$, a detached shock results (see Figures 6.8 and 6.9 in text).

## UNIT 5 - PRANDTL-MEYER FLOW - OBJECTIVES

The student shall be able to:

1. Know how Entropy Changes and Pressure Ratios vary with deflection angles.

2. Explain how finite turns (with finite pressure ratios) can be accomplished isentropically.

3. Show Prandtl-Meyer flow (both expansions and compressions) on a T-S diagram.

4. Demonstrate the development of the relation between MACH number (M) and flow turning angle ($\nu$).

5. Show how tables can be developed for Prandtl-Meyer Flow by the introduction of a reference state.

6. Explain the governing boundary conditions and show the results when shock waves and P-M waves: (a) reflect off physical boundries, and (b) reflect off "free" boundaries.

7. Draw the wave forms created by flow over rounded and/or wedge-shaped wings as the angle of attack changes. Be able to solve problems of this type.

8. Describe and sketch what occurs as fluid flows past a smooth concave corner and a smooth convex corner, explaining why an expansion shock CANNOT occur.

9. Explain with the aid of diagrams the flow conditions at the outlet of a supersonic nozzle created by under-expansion and overexpansion.

10. Solve typical problems using PRANDTL-MEYER flow tables.

11. Demonstrate by a sketch, an understanding of the operating conditions of a fixed-geometry diffuser.

## UNIT 5 - PRANDTL-MEYER FLOW

### 5.1 - Introduction to Prandtl-Meyer Flow

a. You saw from the "Special Problem" that for values of $M_1$ (in a NORMAL shock process) which are close to 1.0, the value of the quantity $(M_1^2 - 1)$, which we will call 'm', is small. It was shown that for small values of 'm', the pressure change across the shock ($\Delta P$) is proportional to the first power of 'm'. We also saw that the entropy change ($\Delta s$) is proportional to the third power of 'm'.

b. Now consider equation *4.5,9 from the last Unit, but inverted.

$$\frac{\tan(\theta-\delta)}{\tan\ \theta} = \frac{(\gamma-1)M_1^2\sin^2\theta + 2}{(\gamma+1)M_1^2\sin^2\ \theta} \qquad (*4.5,9a)$$

We can rearrange this to show:

$$\frac{1}{M_1^2\sin^2\theta} = \frac{(\gamma+1)\tan(\theta-\delta)}{2\tan\ \theta} - \frac{(\gamma-1)}{2} \qquad (*4.5,9b)$$

By applying the auspicious trignometric relations and re ranging we can show that this becomes:

$$M_1^2\sin^2\theta - 1 = \frac{(\gamma+1)}{2}\ M\ \frac{\sin\ \theta\ \sin\ \delta}{\cos(\theta-\delta)} \qquad (*5.1,1)$$

Notice that for small values of $\delta$ where we can approximate $\sin\ \delta$ as $\delta$, and $\cos(\theta-\phi)$ as $\cos\ \theta$:

$$M_1^2\sin^2\theta - 1 \approx \left[\frac{(\gamma+1)}{2}\ M_1^2\ \tan\ \theta\right]\delta \qquad (*5.1,2)$$

You saw from equation *4.5,8 as applied in the "Special Problem":

$$\frac{P_2 - P_1}{P_1} = \frac{2\gamma}{\gamma + 1}\ (M_1^2 - 1) \qquad (*5.1,3)$$

Note that $M_1$ in equation *5.1,3 represents the normal mach number ($M_{1n}$) and since from unit 4.5(b) we saw

$$M_{1n} = M_1 \sin \theta$$

Thus Equation (*5.1,3) as applied to oblique shocks is:

$$\frac{P_2 - P_1}{P_1} = \frac{2\gamma}{\gamma+1} (M_1^2 \sin^2\theta - 1) \qquad (*5.1,4)$$

Thus we can now see that for small deflection angles (and hence weak shocks) combining Equations (*5.1,4) and (*5.1,2) yields:

$$\frac{P_2 - P_1}{P_1} \approx \left[\frac{\gamma M_1^2}{(M_1^2-1)^{1/2}}\right] \delta \qquad (*5.1,5)$$

c. We can draw the conclusion from this that <u>for very weak oblique shocks</u> the pressure change across <u>the shock is proportional to the deflection angle.</u>

In summary: $\Delta P \propto m$ where: $m = (M_{1n}^2 - 1)$

But also $\Delta P \propto m$

and $\Delta s \propto m$

Thus

$$\Delta s \propto \Delta P^3 \propto \delta^3$$

Now suppose we let the deflection angle become infinitely small. What happens to $\Delta s$? If $\Delta s$ is essentially zero for an infintesimally small turn, we can accomplish a finite turn isentropically by a series of such infinitesimally small turns, and the finite turn will have a finite pressure change.

Furthermore, since this situation has no losses, it is <u>reversible</u>. Thus we can have compressions or expansions depending on the boundary conditions.

d. Now read Sections 7.1, 7.2 and 7.3 in the text. Note carefully the reasoning which leads to the Prandtl-Meyer expansion fan pictured in Figure 7.8...and it is ISENTROPIC. In this isentropic (or Prandtl-Meyer) expansion process:

1) Mach number Increases.

2) Mach angle Decreases.

5.2 - Flow Equations

a. Read section 7.4 in the text, through page 125.

b. Notice that the tables are referenced to the boundary condition where $M_o = 1.0$ and $\nu = 0$. Also notice that M is the Mach number AFTER the turn.

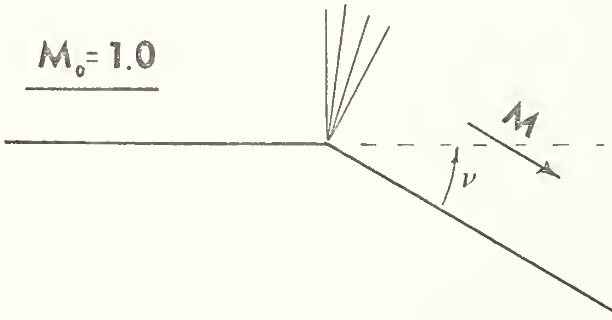



Figure *5.1

c. Now consider Figure 7.10 in the text. Suppose you were given $M_1 = 2.13$ and asked to find $M_2$, given the angle $\nu_2 - \nu_1 = 40°$. From the Prandtl-Meyer flow tables in Appendix D, you can find the "virtual" angle $\nu_1$, which would have produced a Mach number of $M_1 = 2.13$. If we then add this value (30°) to the value of $\nu_1 - \nu_2$, and consult the table for $\nu = 70°$, we find $M_2 = 4.34$.

d. Since this is an isentropic process, we know that $P_{t1} = P_{t2}$ and $T_{t1} = T_{t2}$ , so we may utilize the ISENTROPIC tables (appendix A) to solve for static pressure and temperature.

e. Now read the remainder of Section 7.4 in the text and follow through the examples carefully.

f. Read Section 7.5 in the text. Note that for the compression process the assumption of Prandtl-Meyer flow (hence Isentropic) is only valid in a limited region close to the wall. The discussion on concave corners (Section 7.3 in the text) shows that for many situations involving a smooth concave turn, the flow characteristics away from the boundary or wall can be found by considering the turn as a sharp concave corner. The transition from point to point close to the wall approaches the condition

of a series of Mach waves as shown in Figure 7.5, but as we consider points away from the wall, we find the infinitesmally small compression waves coalesce into a finite oblique shock. Figure 7.6 in the text is not an accurate d piction of the coalescing process. If the area close to the wall is enlarged, as in Figure *5.2.

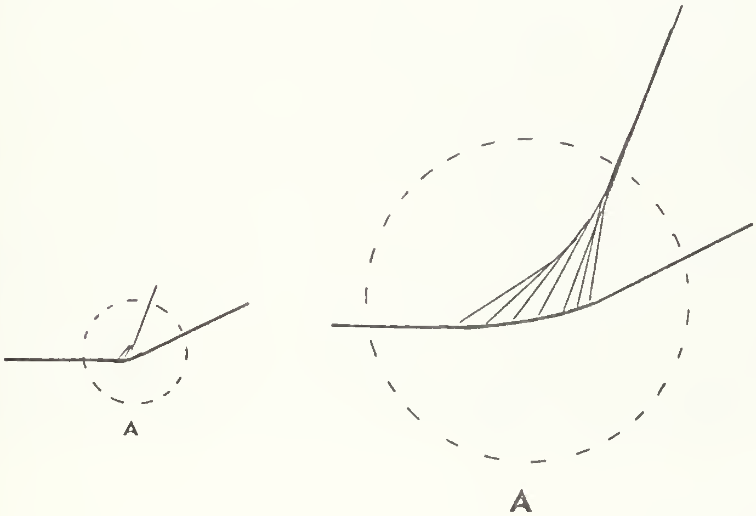



Figure *5.2

We find an envelope of Mach lines forms as the Mach lines coalesce forming a finite shock wave at the Inclination Angle, ($\theta$) corresponding to the overall deflection ($\delta$) and the initial Mach number. Since boundary layer and other real gas effects at the wall are often neglected in basic engineering considerations, we often consider only the flow away from the wall. However if the turn is sufficiently smooth and gradual, the Prandtl-Meyer flow region may be significant, and the flow within a reasonable distance from the boundary may be treated as Prandtl-Meyer flow with reasonable accuracy.

5.3 - Wave Reflections

a. Read Sections 7.6 and 7.7 in the text.

b. Review the summary in Section 6.5 of the text which puts great emphasis on the importance of boundary conditions. Let us now consider the two principle physical conditions that govern wave behavior. We shall accomplish this by analyzing the operation of a converging-diverging axi - symmetric nozzle depicted in Figure *5.3 which is operating between its SECOND and THIRD critical points. Obviously at a physical wall, the flow must be parallel to the wall. We may view the central streamline as though it were a "wall", hence (if we treat the flow as one dimensional we may consider the flow in region 1 to be parallel to the centerline as depicted. For the second condition consider the boundary of the "free jet" as depicted by the dotted line in region 2 . There must be pressure equilibrium along this boundary, so $P_2 = P_{ambient}$. We can now follow from region to region and by matching the required pressures or flow direction angles, we may determine a great deal more about the flow conditions.

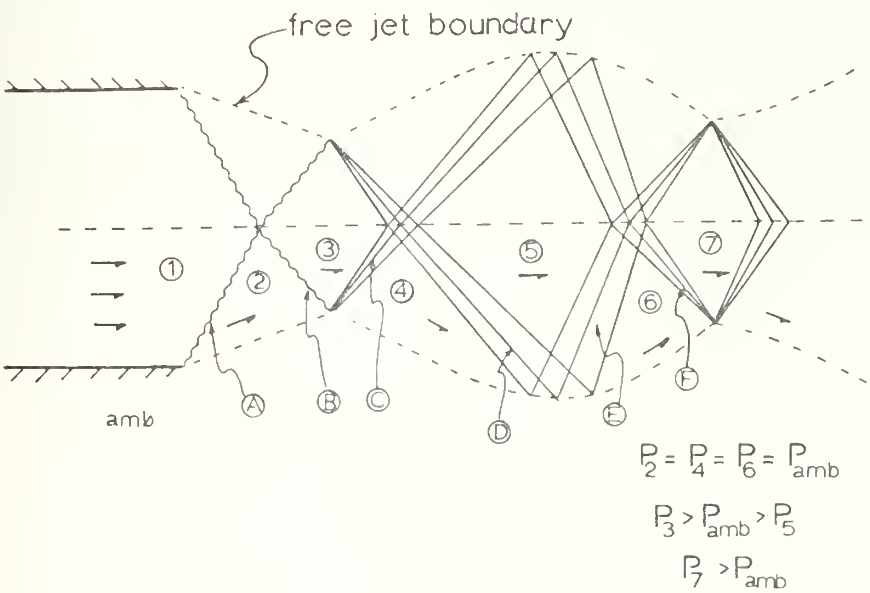



Figure *5.3

c. If we operate this nozzle with a pressure ratio between 2nd and 3rd critical it is obvious that we need a compression process at or beyond the exit in order for the flow to end up at the ambient pressure. However, a normal shock at the exit will produce too strong a compression. What is needed is a shock process that is weaker than a normal shock and the oblique shock has been shown to be just this. Thus, at the exit we observe oblique shock (A) at the appropriate angle so that $P_2 = P_{amb}$.

d. We recall that the flow across an oblique shock is always deflected toward the shock and thus the flow in region (2) is no longer parallel to the centerline. Wave front (B) must deflect the flow back to its original axial direction. This can easily be accomplished by another oblique shock. (an alternate way of viewing this is that the oblique shocks from both upper and lower lips of the nozzle "pass through each other" when they meet at the centerline. If one adopts this philosophy one should realize that the waves are altered in the process of traveling through one another.)

e. Now, since $P_2 = P_{amb}$, passage of the flow through oblique shock (B) will make $P_3 > P_{amb}$ and region (3) can not have a free surface in contact with the surroundings. Consequently a wave formation must emanate from the point where wave (B) meets the free boundary and the pressure must decrease across this wave. An "expansion shock" would fulfill our requirements but we know that no such animal can exist. We now realize that wave form (C) must be a Prandtl-Meyer expansion so that $P_4 = P_{amb}$.

f. However, passage of the flow through the expansion fan (C) causes it to turn away from the centerline. Thus, as each wave of the P-M "expansion fan" meets the centerline a wave form must emanate to turn the flow parallel to the axis again. If wave (D) were a compression in which direction would the flow turn? We see that to meet the boundary condition of flow direction wave (D) must be another P-M expansion. Thus the pressure in region (5) is less than ambient.

g. Can you now reason that to get from (5) to (6) and meet the boundary condition imposed by the free boundary, (E) must consist of P-M compression waves. Similarly (F) must consist of P-M compression waves in order to turn the flow in region (7) to match the direction of the "wall". Now, is $P_7$ equal to, greater than, or less than $P_{amb}$? You should realize that conditions in region (7) are similar to those in region (3) and so the cycle repeats.

h. The situation described above (operation between 2nd and 3rd critical) is referred to as OVEREXPANSION. It is also described in section 8.3 in the text.

i. From this example we may draw some general conclusions about reflections:

   1) Reflections from a Physical (or pseudo-physical boundary where the boundary condition concerns the flow direction) are the same "family". That is, shocks reflect as shocks, compression waves reflect as compression waves, and expansion waves reflect as expansion waves.

   2) Reflections from a "free!" boundary (where pressure equilization occurs) are of the opposite family. i.e., compression waves reflect as expansion waves, and vice versa.

j. The angle through which the flow turns from ③ to ④ is determined by the pressure change required, whereas the angle from ④ to ⑤ depends on the flow in ⑤ paralleling the "wall".

k. Let us now examine an UNDEREXPANDED nozzle. This is operation <u>below</u> 3rd critical.

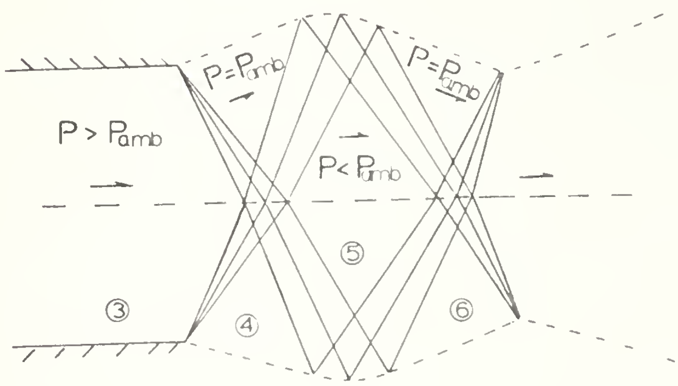



Figure *5.4

Note that the flow leaving this nozzle has a pressure greater than ambient and the flow is parallel to the axis. Reflect back to Figure *5.3 showing the overexpanded nozzle and you will see that the condition is exactly the same as region 3 . Thus the flow patterns are the same from this point on. This situation is also described in Section 8.3 of the text.

l. We should note that at the jet edge in a real gas a SHEAR layer exists as a result of the velocity gradient.

m. Now read Section 7.8 in the text.

## 5.4 - Supersonic Diffuser

a. We will now examine the design of a fixed geometry supersonic diffuser. From our studies of supersonic nozzles we would assume that a converging-diverging section would do the trick - and indeed it will. However, there are some practical operating difficulties that must be considered.

b. Suppose we design the inlet diffuser for an airplane that will fly at Mach 1.86. From the isentropic tables we see that the area ratio corresponding to this Mach No. is 1.507. We shall construct the diffuser with an area ratio (inlet to throat) of 1.500. Now let us follow the operation of this diffuser as the aircraft takes off and accelerated to its design speed. Note that as the flight speed reaches $M_o \approx .43$ the diffuser becomes choked. (Check the subsonic portion of the isentropic tables for the above area ratio.) This condition is shown in Figure *5.5a. Now increase the flight speed to $M_o = .6$. "Spillage" of external diffusion occurs as indicated in Figure *5.5b. As $M_o$ is increased to 1.0 there is a further decrease in the "capture area" (area of the flow at the free stream mach number that actually enters the diffuser).

c. As we increase $M_o$ to supersonic speeds a detached shock wave forms. See figures *5.5d and *5.5e. Note that at the higher flight speeds the shock moves closer to the inlet as less external diffusion is required to reach $M_1 = .43$ at the inlet. Also note that it is necessary to fly at slightly greater than $M_o = 4.19$ in order to have the shock attached to the inlet as shown in figure *5.5f (Check the shock tables to substantiate this.) If we now increase $M_o$ to 4.2 the shock moves very rapidly past the throat and forms in the divergent section downstream of the throat as shown in figure *5.5g. This is referred to as "swallowing the shock" and the diffuser is said to be "started". Under these conditions we no longer have Mach 1.0 in the throat. (Can you compute the Mach number that exists in the throat?) We can now slowly decrease the flight speed to the design condition of $M = 1.86$ and the shock will move to a position just downstream of the throat and occur at a Mach number of just slightly greater than 1.0. (Thus we have a very weak shock and negligible losses.) See figure * 5.5h.

d. Two comments can now be made on the above performance:

1) In order to "start" the diffuser, which was designed for 1.86, it is necessary to "over-speed" the plane to a Mach number of 4.2.

2) If the plane slows just slightly below its design speed — or perhaps minor air disturbance might cause it to drop below 1.86 — the shock will pop out in front of the inlet and the diffuser must be "started" all over again.

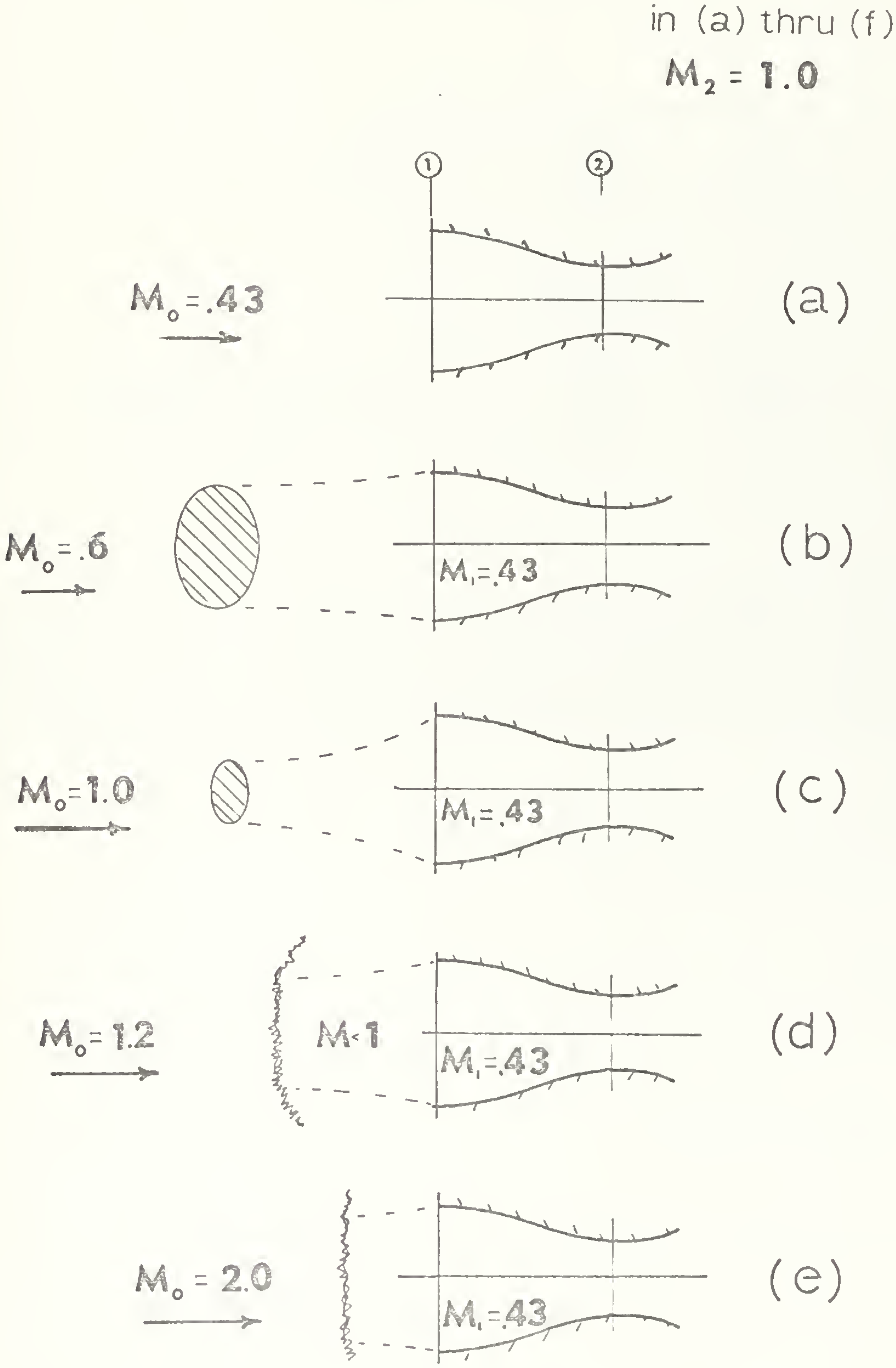
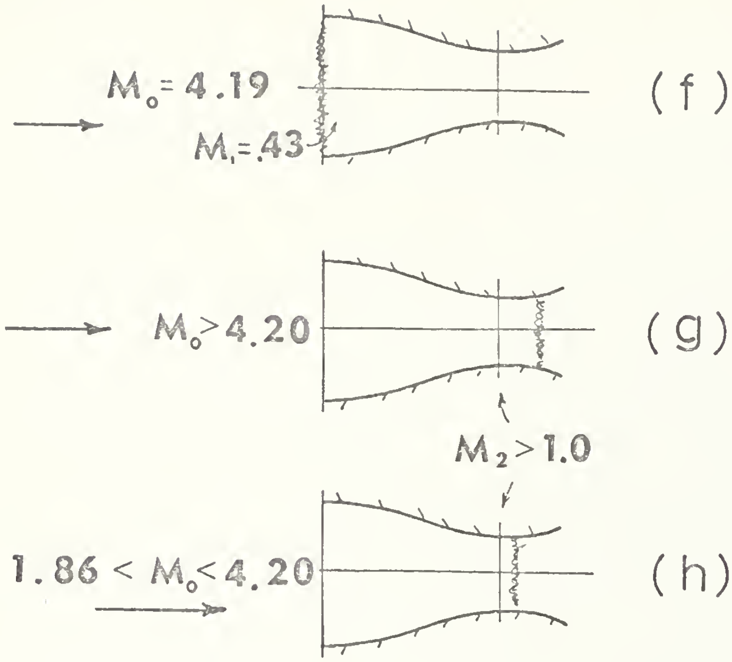



Figure *5.5




e. It should be obvious that either of these situations cannot be tolerated and for this reason one does not see fixed geometry converging-diverging diffusers. We can not summarize the behavior of fixed geometry diffusers as is depicted in figure *5.6.

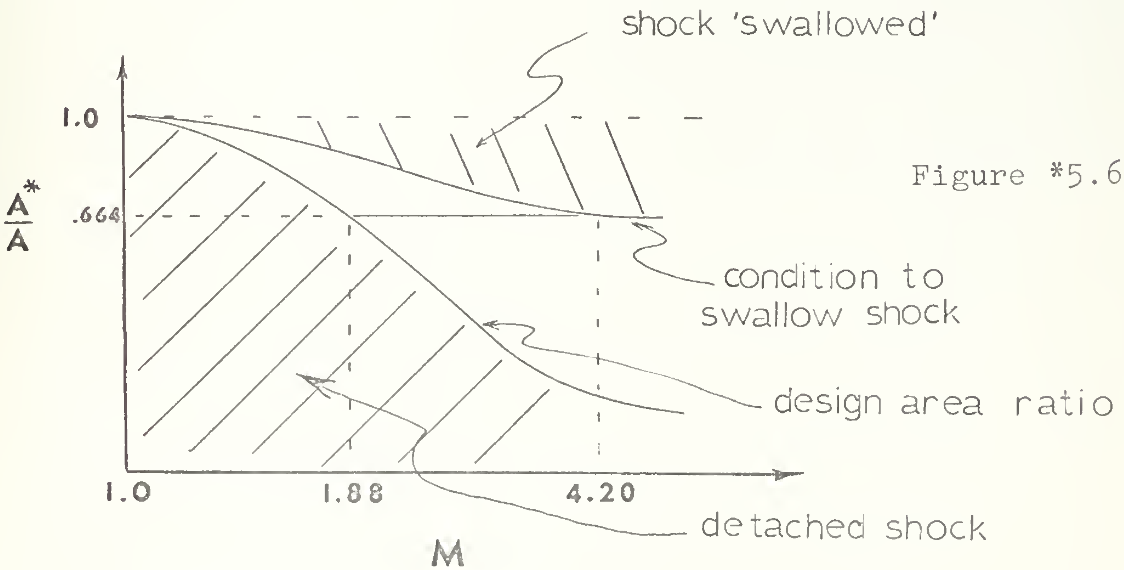



Figure *5.6

5.5 - Airfoils

a. Read Section 8.5 in the text.

b. Follow carefully through the examples in section 8.5.

## UNIT 6 - FANNO FLOW - OBJECTIVES

The student shall be able to:

1. List assumptions and restrictions applicable to FANNO FLOW and relate these to real physical situations.

2. Sketch a Fanno line in the h-s and T-s planes, identifying the regions of sonic, supersonic and sub-sonic flow.

3. Describe the variation of static and stagnation pressure, static and stagnation temperature, density and velocity as flow varies along a FANNO line (for both subsonic and supersonic flow).

4. State what is meant by "choked flow".

5. Describe the effects of changing duct length in a choked Fanno flow situation in supersonic and subsonic flow.

6. Define "friction factor", "equivalent (or hydraulic) diameter", "absolute and relative roughness", "absolute and kinematic viscosity", and "Reynolds number", and know how to determine each.

7. Starting with basic principles, derive expressions for property ratios such as $T_2/T_1$, $P_2/P_1$, etc. in terms of Mach number and specific heats, in a Fanno flow situation. (for Honors credit)

8. Describe (including h-s diagram) how the Fanno tables are developed by use of a reference location.

9. Demonstrate the ability to solve typical Fanno flow problems by use of the appropriate tables and/or relations.

## UNIT 6 - FANNO FLOW - STUDY GUIDE

### 6.1 - Introduction to Fanno Flow

a. We have to this point touched only briefly on the subject of friction losses. This unit will analyze the effects of flow in a constant area duct with friction. The results of this analysis have many direct engineering applications in compressible fluid flow, and will pave the way for increased understanding of the effects of friction on fluid flow under any circumstances. To simplify the analysis we will assume that the flow is adiabatic and no shaft work is added to, or extracted from the flow. This assumption is generally quite valid when dealing with reasonably short ducts where no special attempt is made to transfer heat or work. Additionally, we shall assume steady, one-dimensional flow with no appreciable change in potential. This type of Flow is known as "Fanno Flow".

b. We shall first consider the effect of these assumptions on the basic equations. Then the general variation of fluid properties with Mach number will be considered. Finally, working equations for Fanno flow with a perfect gas will be developed, from which we may see how values are tabulated with respect to a reference condition as has been done with shocks and isentropic flow. Thus we will simplify the business of problem solving. For your own benefit, summarize the assumptions that we are making in Fanno Flow.

## 6.2 - Basic Development of FANNO Flow

a. Consider first the continuity equation for steady one-dimensional flow as developed in Unit 1:

$$\dot{m} = \rho A V = ¢ \qquad (*6.2,1)$$

Since the cross-sectional area is constant, this reduces to:

$$\rho V = ¢ \qquad (*6.2,2)$$

We will assign a new symbol, "G" to the quantity '$\rho V$' which is referred to as the "mass-velocity". So by definition:

$$\rho V \equiv G \qquad (*6.2,3)$$

What are the units of "G"?

b. We will now resurrect the energy equation, which for steady flow between any two points in the flow may be written in terms of stagnation enthalpy:

$$h_{t_1} + q_{1\to 2} = h_{t_2} + w_{1\to 2} \qquad (*2.1,7)$$

but since

$$q_{1\to 2} = w_{1\to 2} = 0$$

then

$$h_{t_1} = h_{t_2} = h_t = ¢ \qquad (*6.2,4)$$

We can write this equation in terms of static conditions as:

$$h_t = h + \frac{V^2}{2g_c} + \frac{gz}{g_c} = ¢ \qquad (*2.1,4)$$

Now assuming the potential remains essentially constant and utilizing equation *6.2,3 we can rewrite equation *2.1,4 as:

$$h + \frac{G^2}{\rho^2 2g_c} = h_t = \text{¢} \qquad (*6.2,5)$$

Since for a given flow, G and $h_t$ are constant, we see that for Fanno flow h and $\rho$ are uniquely related. We can now plot this relationship for various values of G (these are called Fanno Lines) in the h-v plane (recalling that $v = 1/\rho$):

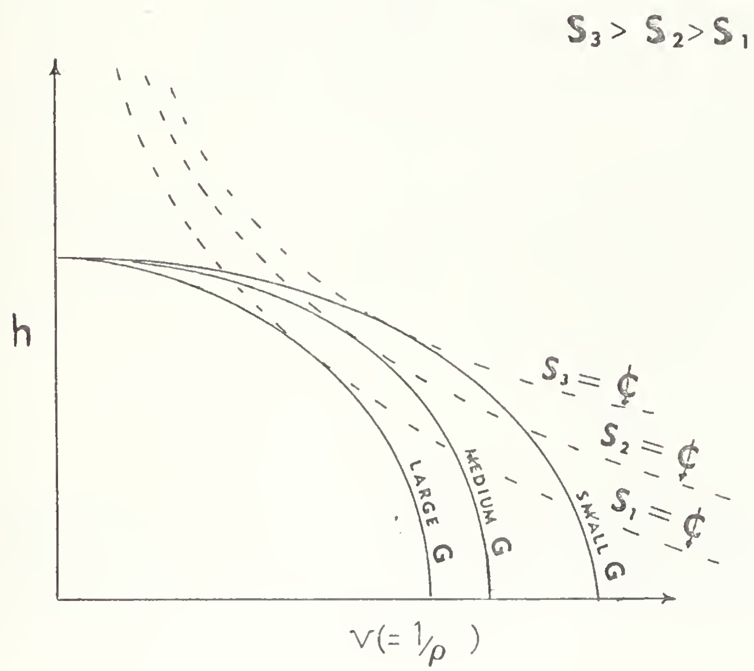



Figure *6.1

If the fluid is known, one can also plot, as in Figure *6.1, lines of <u>constant entropy</u> (dotted lines) which come from thermodynamic considerations. We can then replot the "Fanno lines" in the more familiar h-s plane as shown in Figure *6.2.

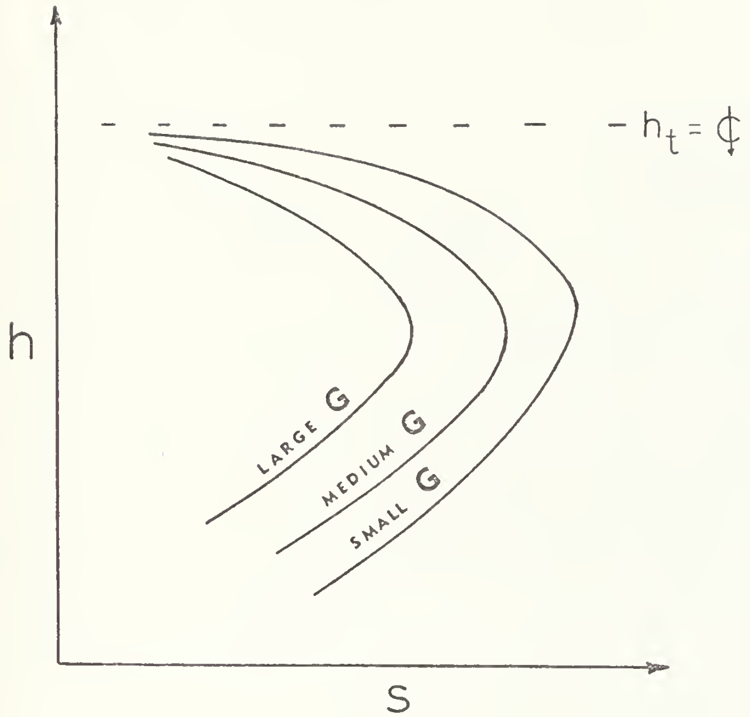



Figure *6.2

c. We may now state in general that the combination of the continuity equation with the energy equation and the thermodynamic equation of state (which established the lines of constant entropy) form the locus of points on a given Fanno Line.

One factor stands out:

> Since this is ADIABATIC, i.e., $dS_e = 0$, the only way entropy can be generated is by LOSSES, so a process can only move toward INCREASING values of S.

6.3 - FANNO FLOW versus MACH NUMBER

a. We shall note that due to the irreversible nature of FANNO Flow, a FANNO LINE consists of two distinct regions separated by a Limiting Point.

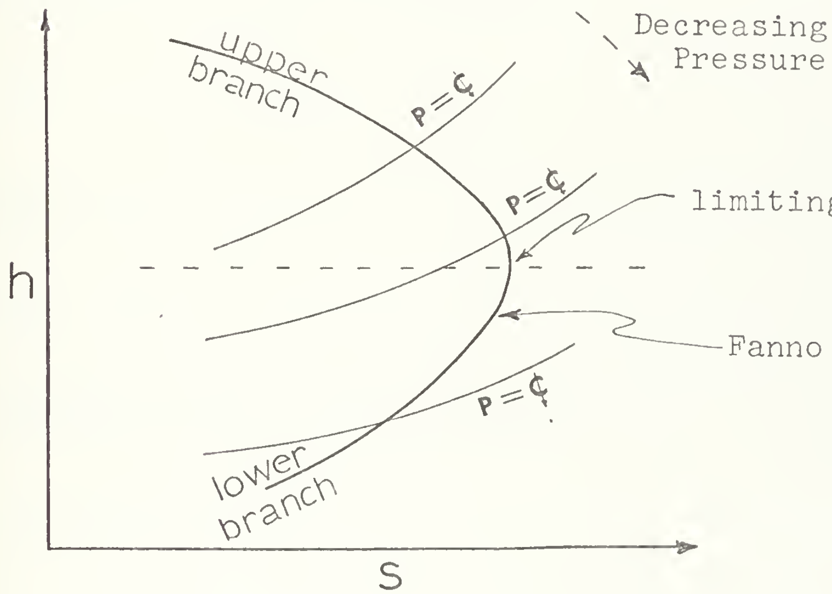



Figure *6.3

We note that in the upper branch a process (which must always result in INCREASING ENTROPY) results in DECREASED PRESSURE. From Figure *6.1, we say what as enthalpy decreases, so must density decrease, hence velocity must INCREASE!

b. Now consider for yourself what happens on the LOWER branch. Draw an arrow on the Lower Branch of Figure *6.3 to indicate which way a process must proceed. As we move along this branch of the Fanno line:

Enthalpy ____________________

Density ____________________

Velocity ____________________

Pressure ____________________

c. Let us now consider the <u>Limiting Point</u>. We are seeking to explore a condition where $ds = 0$. Recalling the energy equation (neglecting Potential changes).

$$h_t = h + \frac{V^2}{2g_c} = ¢ \qquad (*2.1,4)$$

Differentiate this to obtain:

$$0 = dh + \frac{VdV}{g_c} \qquad (*6.3,1)$$

Also recall that

$$\rho V = ¢ \qquad (*6.2,2)$$

Differentiating this we obtain:

$$\rho dV + Vd\rho = 0 \qquad (*6.3,2a)$$

or

$$dV = -\frac{Vd\rho}{\rho} \qquad (*6.3,2b)$$

Substitute equation *6.3,2b into Equation *6.3,1 and solve for dh:

$$dh = \frac{V^* d\rho}{g_c} \qquad (*6.3,3)$$

Recall the thermodynamic property relation

$$Tds = dh - vdp \qquad (*6.3,4a)$$

$$Tds = dh - \frac{dp}{\rho} \qquad (*6.3,4b)$$

Combining Equations *6.3,3 and *6.3,4b we find that

$$\boxed{Tds = \frac{V^2 d\rho}{g_c \rho} - \frac{dp}{\rho}} \qquad (*6.3,5)$$

This expression is valid at ANY point on a Fanno Line. We now apply equation *6.3,5 to the limiting point where ds = 0, then we find at the limiting point:

$$\frac{dp}{\rho} = \frac{V^2 d\rho}{g_c \rho}$$

or

$$V^2 = g_c \left.\frac{dp}{d\rho}\right|_{\substack{\text{At}\\ \text{Limit}\\ \text{Point}}} = g_c \left.\frac{\partial p}{\partial \rho}\right)_{s=\text{constant}}$$

We recognize this as the expression for $a^2$ (see section 2.3 in the text). Hence at the limiting point:

$$V = a$$

or

$$M = 1.0$$

## 6.4 - WORKING RELATIONS for FANNO FLOW in a PERFECT GAS

a. From the continuity equation in steady, one-dimensional flow in a constant area duct:

$$G = \rho V = \not{c} \qquad (*6.2,2)$$

From the Perfect Gas Law:

$$\rho = \frac{P}{RT}$$

and from the definition of Mach number:

$$V = Ma$$

and for a Perfect Gas

$$a = \sqrt{\gamma g_c RT}$$

Equation (*6.2,2) can be re-written:

$$\frac{PM}{\sqrt{\frac{RT}{\gamma g_c}}} = \not{c} \qquad (*6.4,1)$$

b. Now we turn to the Energy Equation. We showed that between two points in a Fanno Line:

$$h_{t_1} = h_{t_2} \qquad (*6.4,2)$$

Recall that for a perfect gas:

$$h_t = c_p T_t \qquad (*6.4,3)$$

So

$$T_{t_1} = T_{t_2} = T_t = ¢ \qquad (*6.4,4)$$

We also saw that for a Perfect Gas:

$$T_t = T(1 + \frac{\gamma-1}{2} M^2) \qquad \text{(3.6) in text}$$

so between two sections

$$T_1(1 + \frac{\gamma-1}{2} M_1{}^2) = T_2(1 + \frac{\gamma-1}{2} M_2{}^2) \qquad (*6.4,5a)$$

or as a temperature ratio

$$\frac{T_2}{T_1} = \frac{1 + \frac{\gamma-1}{2} M_1{}^2}{1 + \frac{\gamma-1}{2} M_2{}^2} \qquad (*6.4,5b)$$

c. By expressing the relation in Equation *6.4,1 between two points in the flow we can write

$$\frac{P_1M_1}{\sqrt{T_1}} = \frac{P_2M_2}{\sqrt{T_2}} \qquad (*6.4,6a)$$

This can be restated as a ratio of the <u>static pressures</u>:

$$\frac{P_2}{P_1} = \frac{M_1}{M_2}\left(\frac{T_2}{T_1}\right)^{1/2} \qquad (*6.4,6b)$$

or, utilizing equation *6.4,5b we can now obtain an expression for a ratio of the static pressures in terms of $\gamma$ and Mach Number:

$$\frac{P_2}{P_1} = \frac{M_1}{M_2} \left[ \frac{1 + \frac{\gamma-1}{2} M_1^{\,2}}{1 + \frac{\gamma-1}{2} M_2^{\,2}} \right]^{1/2} \qquad (*6.4,7)$$

Have you seen some of these equations before? For what kind of flow? What assumptions have you fed into these equations so far?

d. Now recall the property relation

$$Tds = dh - \frac{dP}{\rho}$$

What restrictions, if any, are attached to this expression? If we now recall from Section 1.11 in the text that:

$$dh = c_p dT \qquad (*6.4,8)$$

and

$$C_p = R \left[ \frac{\gamma}{\gamma-1} \right] \qquad (*6.4,9)$$

we can show that

$$Tds = \left[ \frac{\gamma}{\gamma-1} \right] RdT - \frac{dP}{\rho} \qquad (*6.4,10a)$$

and we can rearrange this to show

$$\frac{ds}{R} = \left[ \frac{\gamma}{\gamma-1} \right] \frac{dT}{T} - \frac{dP}{P} \qquad (*6.4,10b)$$

Now integrate Equation *6.4,10b between two points in the flow to show that

$$\frac{S_2-S_1}{R} = \frac{\gamma}{\gamma-1} \ln \frac{T_2}{T_1} - \ln \frac{P_2}{P_1} \qquad (*6.3,10c)$$

Utilizing the expressions we have just developed for the static pressure ratio and the static temperature ratio we can now express the entropy change as a function of $\gamma$ and Mach Number:

$$\frac{S_2-S_1}{R} = \frac{\gamma}{\gamma-1} \ln \left[ \frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2} \right] - \ln \left[ \frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2} \right]^{1/2} \qquad (*6.4,11a)$$

You can now combine the terms of this expression to show that

$$\frac{S_2-S_1}{R} = \ln \frac{M_2}{M_1} \left[ \frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2} \right]^{\frac{\gamma}{\gamma-1} - \frac{1}{2}} \qquad (*6.4,11b)$$

or

$$\frac{S_2-S_1}{R} = \ln \frac{M_2}{M_1} \left[ \frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2} \right]^{\frac{\gamma+1}{2(\gamma-1)}} \qquad (6.4,11c)$$

Refer back to the section where we developed the stagnation pressure ratio as:

$$\frac{P_{t_2}}{P_{t_1}} = e^{-\frac{\Delta s}{R}} \qquad (*3.3,15)$$

You should satisfy yourself that this relation applies to Fanno Flow.

If we now convert Equation *6.4,11c to:

$$-\frac{S_2 - S_1}{R} = \ln \frac{M_1}{M_2}\left[\frac{1+\frac{\gamma-1}{2}M_2^{\;2}}{1+\frac{\gamma-1}{2}M_1^{\;2}}\right]^{\frac{\gamma+1}{2(\gamma-1)}} \qquad (*6.4,12)$$

Now take the anti-log of both sides to obtain

$$e^{-\frac{\Delta s}{R}} = \frac{P_{t_2}}{P_{t_1}} = \frac{M_1}{M_2}\left[\frac{1+\frac{\gamma-1}{2}M_2^{\;2}}{1+\frac{\gamma-1}{2}M_1^{\;2}}\right]^{\frac{\gamma+1}{2(\gamma-1)}} \qquad (*6.4,13)$$

We now have the means to obtain all the properties at a downstream point 2 if we know all the properties at some upstream point 1 and the Mach Number at point 2. But we need to be able to predict the Mach Number at 2 from the physical set-up of the flow (i.e., Duct length, etc.).

e. In Unit 1, we developed an expression for the Momentum Equation as it applies to steady, one-dimensional flow in any fluid

$$\frac{dP}{\rho} + \frac{fdxV^2}{D_e 2g_c} + \frac{g}{g_c}dz + \frac{VdV}{g_c} = 0 \qquad (*1.6,16)$$

If we now apply the perfect gas law and the definition of Mach Number to this equation, we can show (neglecting Potential change):

$$\frac{dP}{P}(RT) + \frac{fdx}{D_e}\frac{M^2\gamma g_c RT}{2g_c} + \frac{M\sqrt{\gamma g_c RT}\,dV}{g_c} = 0 \qquad (*6.4,14)$$

This can be rewritten as:

$$\frac{dP}{P} + \frac{f dx}{D_e} \frac{M^2 \gamma}{2} + \underbrace{\frac{\gamma}{2} dM^2 + \frac{\gamma}{2} M^2 \frac{dT}{T}} = 0 \qquad (*6.4,15)$$

These two terms result from eliminating dV in the last term of Equation *6.4,14. <u>Can you show this</u>?

f. We must first evaluate dP/P as a function of Mach Number. From continuity:

$$\frac{PM}{\sqrt{T}} = \text{¢} \qquad (*6.4,6)$$

Therefore:

$$\ln P + \ln M - \frac{1}{2} \ln T = \ln \text{¢} \qquad (*6.4,16a)$$

Differentiate to show:

$$\frac{dP}{P} + \frac{dM}{M} - \frac{1}{2} \frac{dT}{T} = 0 \qquad (*6.4,16b)$$

Now we must obtain an expression for dT/T as a function of Mach number. From the <u>energy</u> equation we obtained:

$$T_t = T(1 + \frac{\gamma-1}{2} M^2) = \text{¢}$$

or

$$\ln T + \ln (1 + \frac{\gamma-1}{2} M^2) = \ln \text{¢} \qquad (*6.4,17a)$$

Again differentiate to obtain

$$\frac{dT}{T} + \frac{d(1 + \frac{\gamma-1}{2} M^2)}{1 + \frac{\gamma-1}{2} M^2} = 0 \qquad (*6.4,17b)$$

Now combine equation *6.4,17b with equation *6.4,16b and substitute into the momentum equation in the form of equation *6.4,15 to show:

$$\frac{fdx}{D_e} = \left(\frac{\gamma+1}{2\gamma}\right) \frac{d(1 + \frac{\gamma-1}{2} M^2)}{1 + \frac{\gamma-1}{2} M^2} + \frac{2dM}{\gamma M^2} - \left(\frac{\gamma+1}{2\gamma}\right) \frac{dM^2}{M^2} \qquad (*6.4,18)$$

We can now integrate equation *6.4,18 from point (1) to point (2) in the flow (see Figure *6.4) to obtain:

$$\frac{f(x_2-x_1)}{D_e} = \frac{\gamma+1}{2\gamma} \ln \left[\frac{1 + \frac{\gamma-1}{2} M_2{}^2}{1 + \frac{\gamma-1}{2} M_1{}^2}\right] - \left[\frac{1}{\gamma}\right]\left[\frac{1}{M_2{}^2} - \frac{1}{M_1{}^2}\right] - \frac{\gamma+1}{2\gamma} \ln \frac{M_2{}^2}{M_1{}^2}$$

(*6.4,19)

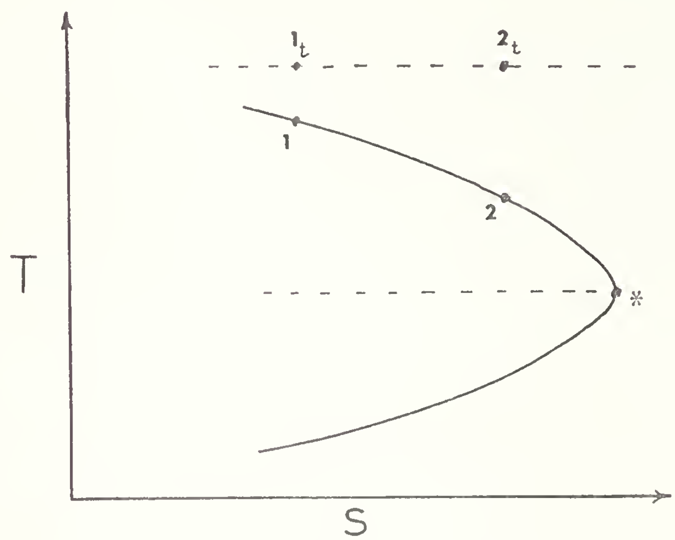



Figure *6.4a

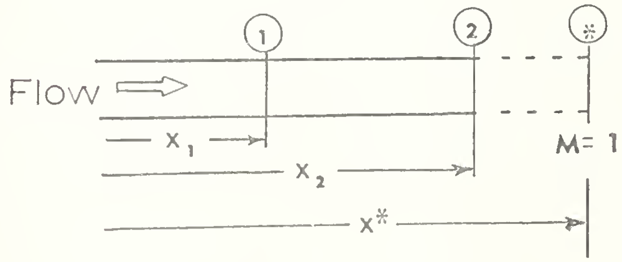



Figure *6.4b

Knowing the duct length $(x_2 - x_1)$, flow rate, fluid and duct material (hence the friction factor), configuration (to obtain $D_e$) and all conditions at one point, we may now find the conditions at the other point.

g. The solution of problems (as in previous analyses) is simplified by the introduction of a reference condition. The obvious choice is the <u>limiting condition</u> where M = 1.0. If we let point 1 be the "*" condition (i.e., M = 1.0; <u>reached by Fanno Flow</u>) then:

$$T_1 = T^*$$

$$P_1 = P^*$$

Point 2 may be any other point in the flow. Equation *6.4,5b may now be reduced to:

$$\frac{T}{T^*} = \frac{\frac{\gamma+1}{2}}{1 + \frac{\gamma-1}{2} M^2} \qquad (*6.4,20)$$

Using a similar approach, obtain the expressions for:

$$\frac{P}{P^*} = \qquad ? \qquad (*6.4,21)$$

and

$$\frac{P_t}{P_t^*} = \qquad ? \qquad (*6.4,22)$$

You should also be able to corroborate that

$$\frac{\rho}{\rho^*} = \frac{V^*}{V} = \frac{1}{M}\left[\frac{2(1 + \frac{\gamma-1}{2} M^2)}{\gamma + 1}\right]^{1/2} \qquad (*6.4,23)$$

h. Now check your expressions for $P/P^*$ and $P_t/P_t{}^*$ by solving for any arbitrary value of Mach (subsonic or supersonic) and using a value of $\gamma = 1.4$. Then compare your solutions against the tabularized values in Appendix E in the text.

## 6.5 - Applications of Fanno Flow

a. Applying the reference condition to equation *6.4,19 (i.e., $M_1 = 1$, $M_2 = M$) yields:

$$\frac{f(x-x^*)}{D_e} = \frac{\gamma+1}{2} \ln \left[\frac{1 + \frac{\gamma-1}{2} M^2}{\frac{\gamma+1}{2}}\right] - \frac{\gamma+1}{2} \ln M^2 - \frac{1}{\gamma}\left[\frac{1}{M^2} - 1\right] \qquad (*6.4,24)$$

Since x* is always greater than x (see Figure *6.4b), it makes sense to change all signs in Equation *6.4,24 to obtain a positive expression which we can simplify to:

$$\frac{f(x^*-x)}{D_e} = \frac{\gamma+1}{2\gamma} \ln\left[\frac{\frac{\gamma+1}{2} M^2}{1 + \frac{\gamma-1}{2} M^2}\right] + \frac{1}{\gamma}\left[\frac{1}{M^2} - 1\right] \qquad (*6.4,25)$$

The quantity (x*-x) now represents the maximum duct length which may be added to the given duct and still maintain the same flow. This expression

$$\frac{f\ L_{max}}{D_e}$$

is tabulated in Appendix E in the text.

b. In order to solve problems we must be able to calculate friction factor (f) and equivalent (or "hydraulic") diameter ($D_e$). From Unit 1 you may recall the definition of these two quantities.

$$f \equiv \frac{4\tau_f}{\frac{\rho V^2}{2g_c}}$$

$$D_e \equiv \frac{4A}{\text{P}}$$

For a circular duct, $D_e$ is simply the diameter (D).

c. The friction factor may seem a bit intractible since the shear force ($\tau_f$) is difficult to measure. However, you may recall from your previous courses in Fluid Mechanics that friction factor is a function of Reynolds number ($N_R$) and "relative roughness" which we can define as $\varepsilon/D_e$ where "$\varepsilon$" is an average value for the absolute roughness of a given material. Table *6.1 gives some typical values of "$\varepsilon$" (which has the dimension of "feet") for various materials. Thus the relative roughness ($\varepsilon/D_e$) is a dimensionless parameter.

TABLE *6.1

TYPICAL ROUGHNESS VALUES

| Material | "$\varepsilon$" (in 'feet') |
|---|---|
| glass, brass, copper, lead | smooth < .0001 |
| steel, wrought iron | .00015 |
| asphalted cast iron | .0004 |
| galvanized iron | .0005 |
| cast iron | .00085 |
| wood staves | .002 |
| concrete | .01 |
| riveted steel | .03 |

d. Similarly, Reynolds number is a dimensionless parameter which relates size and viscosity. Reynolds number is defined as:

$$N_R = \frac{\rho V D}{\mu g_c}$$

where $\mu$ is "absolute viscosity"

If we define a "kinematic viscosity"

$$\nu = \frac{\mu g_c}{\rho}$$

Then

$$N_R = \frac{VD}{\nu}$$

Table 3 on page 387 in the text gives typical values for "$\mu g_c$" for various gases. With this information, and knowing the conditions at a given point in the flow, we can calculate the density and velocity (remember $G = \rho V = \text{¢}$) hence Reynolds number. Then using experimentally derived curves such as depicted in Figure *6.5, the friction factor may be determined.

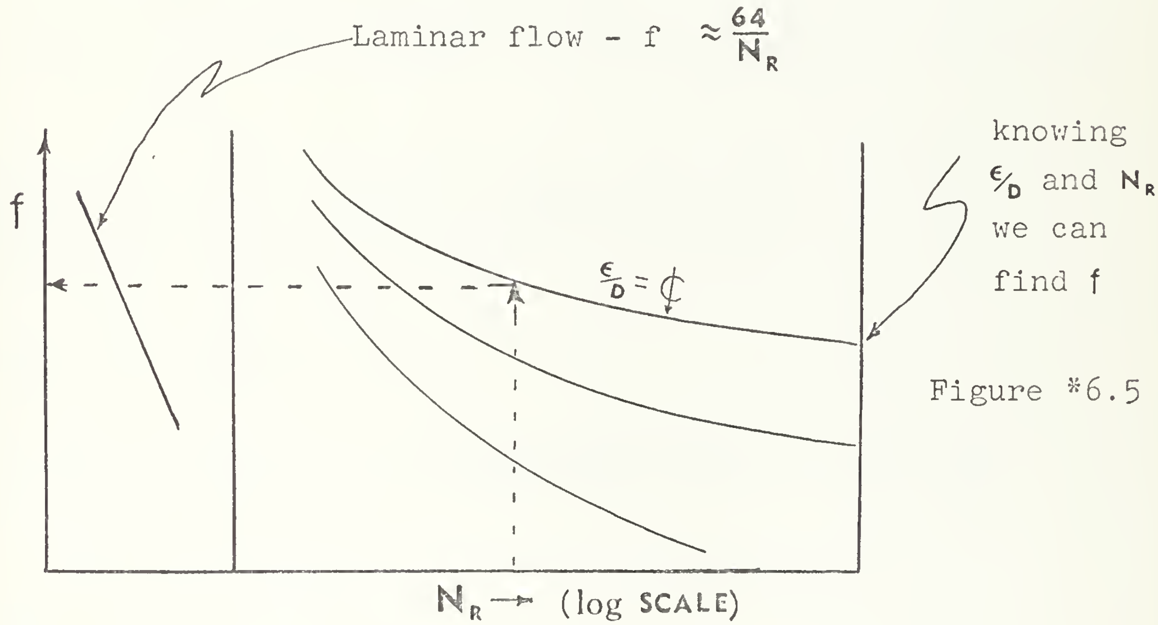



Figure *6.5

e. Now read sections 9.1 and 9.2 in the text. Study carefully example 9.1.

f. Now consider the following example problem:

Given a circular <u>duct</u> with these specifications

Diameter - 6 inches

Length - 100 feet ($\Delta x$)

Material - Galvanized Iron

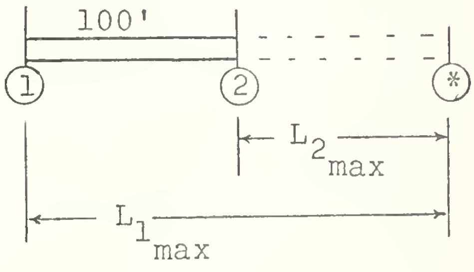



Figure *6.6

It is desired to have this duct deliver 800 cubic feet/minute of air at 70°F and 18 psia. Find the inlet conditions required at point (1).

First, let us summarize a step-by-step problem solution technique:

1) Sketch the physical problem.
2) Establish and designate locations where conditions are known and/or desired.
3) Compute the Equivalent Diameter.
4) Find friction factor.
5) From equation *6.4,19 (or the Tables using $FL_{max}/D$, if known) find Mach number(s).
6) Use relations or Tables to find additional properties desired.

Now let us apply this approach to our example problem. Steps 1) and 2) are done in Figure *6.6.
Recall for a circular duct that the Equivalent Diameter is the pipe diameter.

$$D_e = .5 \text{ feet}$$

From the Table *6.1 for <u>Galvanized Iron</u> the value for $\varepsilon/D$ is

$$\frac{\varepsilon}{D} = \underline{\qquad\qquad\qquad}_{?} .$$

To find Reynolds number we must know $\rho$ and V. Since we know the conditions at (2), we can compute

$$M_2 = \frac{V_2}{a_2}$$

$$a_2 = \sqrt{\gamma g_c R\ T_2} = 49\sqrt{540} = 1128 \text{ ft/sec}$$

$$\rho_2 = \frac{P_2}{R\ T_2} = .0918 \text{ lbm/ft}^3$$

$$A_2 = \frac{\pi D^2}{4} = \frac{\pi}{16} \text{ ft}^2$$

$A_2V_2$ = Volumetric flow rate

So

$$V_2 = \underset{?}{\underline{\qquad\qquad\qquad}} \quad \text{ft/sec}$$

$$M_2 = \frac{V_2}{a_2} = .602 \approx .60$$

$$N_R = \underset{?}{\underline{\qquad\qquad\qquad}}$$

$$\frac{\epsilon}{D} = \underset{?}{\underline{\qquad\qquad\qquad}}$$

Now you should be able to determine from the attached "Friction Factor for Pipes" chart:

$$f = .0197 \approx .02$$

So

$$\frac{f\Delta x}{D} = \frac{(.02)(100)}{(.5)} = 4.00$$

From the Appendix E, for $M_2 = .6$, $\frac{f\ L_{max}}{D} = .49081$

thus

$$\frac{f\ L_1}{D} = \frac{f\ L_2}{D} + \frac{f\Delta x}{D} = \underset{?}{\underline{\qquad\qquad\qquad}}$$

Now consult Appendix E to find $M_1 = \underset{?}{\underline{\qquad\qquad\qquad}}$

Now knowing $M_1$ and $M_2$ you can use the property ratios to obtain the properties at point (1).

$$T_1 = \frac{T_1}{T^*} \frac{T^*}{T_2} T_2 = 556\ °R \quad ; \quad P_1 = \underset{?}{\rule{4cm}{0.4pt}}$$

Note that we do not have to bother with subscripts on the * quantities (such as T* and P*) since for any given Fanno Line there is only one reference location.

g. Now read section 9.3 in the text. Pay close attention to the flow situations as described in Figure 9.16. We will not delve into the subject of Iso-Thermal Flow, but if time permits, you are encouraged to finish reading Chapter 9 in the text.

## Unit 7. - RAYLEIGH FLOW - OBJECTIVES

The student shall be able to:

1. State the assumptions and restrictions utilized in the analysis of constant area flow with simple heat transfer (Rayleigh Flow).

2. Given the general equations of energy, continuity, momentum and state, demonstrate how the assumptions in objective (1) modify or eliminate each term.

3. Sketch a Rayleigh line in the P-v plane together with lines of constant entropy and lines of constant temperature. Indicate the points of maximum temperature and maximum entropy, and the directions of increasing entropy and temperature.

4. Sketch a Rayleigh line on a T-s diagram, indicating the regions of Sonic, Subsonic and Supersonic flow.

5. Describe the changes in fluid properties which occur as one moves along a Rayleigh line for the case of heating and the case of cooling in both subsonic and supersonic flow.

6. Correlate the reference points of maximum temperature and maximum entropy on a T-s diagram of a Rayleigh line with those on a P-v diagram.

7. Explain what is meant by "thermal choking".

8. Explain by a T-s diagram how the flow adjusts to the addition of heat in a constant area duct in a "thermally choked" flow situation in both subsonic and supersonic flow.

9. Solve typical Rayleigh flow problems by the use of tables and equations.

## Unit 7. - RAYLEIGH FLOW - STUDY GUIDE

### 7.1 Introduction to Rayleigh Flow

a) Constant area flow with significant heat transfer, addition or removal, is called "Rayleigh flow". We will approach the analysis of this flow condition in a manner similar to that used in previous flows such as normal shocks, Fanno flow, etc. We will first develop the basic equations as applied to the Rayleigh flow situation. Then perfect gas relations will be introduced. Finally, we will introduce a reference state which permits tables to be developed, which will in turn simplify problem solution techniques.

b) Read Section 10.1 in the text.

## 7.2 - Basic Relations

a. We can summarize the assumptions made in Rayleigh flow as follows:

1) Steady flow
2) One-dimensional flow
3) Negligible change in potential energy
4) No work (shaft)
5) Constant area
6) Negligible friction

This last assumption warrants some comment since internal irreversible effects ($dS_i$) are obviously present. What is really being said is that since we are dealing with significant heat transfer the change in entropy from this cause ($dS_e$) will be relatively large. Thus the assumption that is actually being made is that:

$$dS_e >> dS_i$$

and we can thus neglect $dS_i$ and say that

$$dS_e \approx dS \qquad (*7.2,1)$$

b. From the continuity equation, we will recall

$$\rho AV = \dot{m} = \text{¢}$$

but $A = \text{¢}$ hence we can say

$$\rho V = \text{¢}$$

Do you recall this expression? Do we have a name for this constant?

c. The energy equation for steady, one-dimensional flow can be written as:

$$h_{t_1} + q_{1\to 2} = h_{t_2} + w_{s_{1\to 2}} \qquad (*7.2,2)$$

For our flow $s_{1\to2} = 0$ , and

$$h_{t_1} + q_{1\to2} = h_{t_2} \qquad (*7.2,3)$$

> A Note of Caution:
> This is the first major flow category for which '$h_t$' has not been constant. Consider a perfect gas. Will '$T_t$' be constant?

d. Recall our basic momentum equation in differential form:

$$\frac{dP}{\rho} + f\,\frac{dx}{D_e}\,\frac{V^2}{2g_c} + \frac{VdV}{g_c} + \frac{g}{g_c}\,dz = 0 \qquad (*7.2,4)$$

Under our assumptions this becomes:

$$\frac{dP}{\rho} + \frac{VdV}{g_c} = 0 \qquad (*7.2,5a)$$

or

$$dP + \rho\,\frac{VdV}{g_c} = 0 \qquad (*7.2,5b)$$

From continuity we know that $\rho V = G =$ constant. Thus the momentum equation can be written as

$$dP + \frac{GdV}{g_c} = 0 \qquad (*7.2,6)$$

This can easily be integrated to give:

$$P + \frac{GV}{g_c} = 0 \qquad (*7.2,7)$$

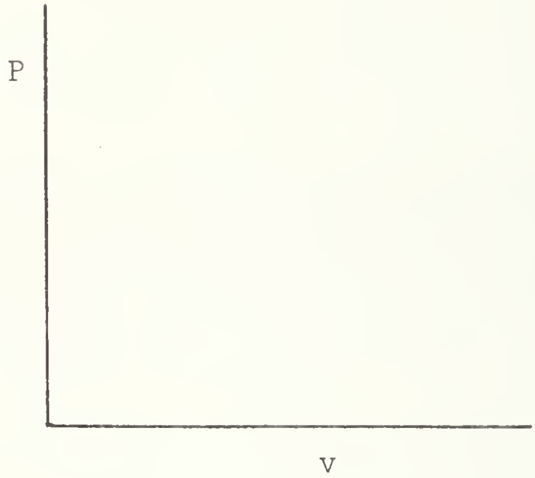
Or, by substituting for the velocity, this can be written as

$$P + \frac{G^2}{g_c \rho} = \text{Constant} \qquad (*7.2,8)$$

or

$$P + \frac{G^2}{g_c} v = \text{Constant} \qquad (*7.2,9)$$

For a given flow rate this indicates a linear relationship between P and v. Try sketching equation (*7.2,9) in the P-v plane. This line is called a "Rayleigh line."




Hint:

$y = mx + b$

Figure *7.1

e. As an aside, rewrite equation (*7.2,7) as:

$$PA + \frac{(\rho A V)\ V}{g_c} = \text{Constant}$$

or

$$PA + \frac{\dot{m}\ V}{g_c} = \text{Constant}$$

Compare this expression with equation (4.2,5) in the unit on shocks. This constant is sometimes called the "impulse function" and is given the symbol "F" or "I" by various authors.

f. If we add lines of constant temperature to our plot (this is easy if we are dealing with a perfect gas) we will have a diagram as shown in Figure *7.2.

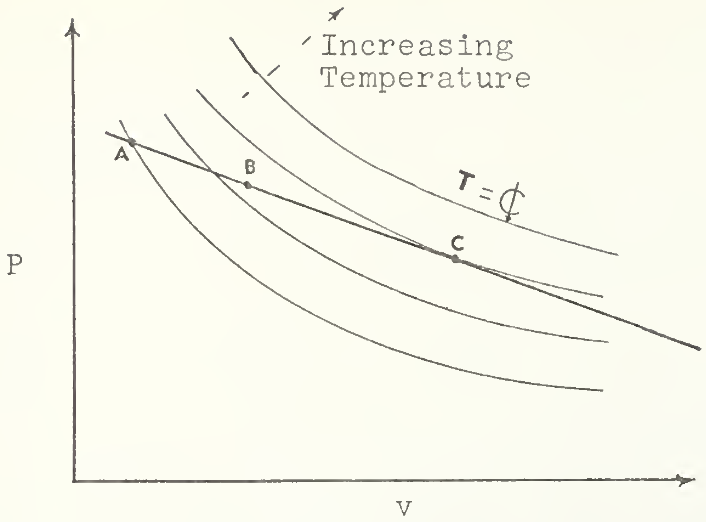



Figure *7.2

Consider a heating process which moves from A to B along the Rayleigh line. If we add more heat, we move farther along and the temperature increases more. When point C is reached, the fluid has reached a maximum temperature. Can we move farther along this Rayleigh line? What is the limiting factor?

Recall that as we add heat we are increasing the entropy of the fluid ($dS_e = dq/T$). Thus we had better investigate lines of constant entropy. When these lines are plotted on Figure *7.2 they are very similar to the lines of constant temperature. To get an accurate picture of what this plot will look like we must investigate the relative slopes of 'T = Constant' and 'S = Constant' lines in the P-v diagram. It will be much easier to carry out this investigation if we consider perfect gases.

For a 'T = Constant' line

$$Pv = RT = \text{constant}$$

$$Pdv + vdP = 0$$

and

$$\frac{dP}{dv} = -\frac{P}{v} \qquad (*7.2,10)$$

For an 'S = Constant' line

$$Pv^{\gamma} = \text{constant}$$

$$v^{\gamma}dP + P\gamma v^{\gamma-1}dv = 0$$

and

$$\frac{dP}{dv} = - P\gamma \frac{v^{\gamma-1}}{v^{\gamma}} = - \gamma \frac{P}{v} \qquad (*7.2,11)$$

Comparing equations (*7.2,10) and (*7.2,11) and noting that $\gamma$ is always greater than 1.0, we see that the 'S = constant' line has a greater negative slope and thus these lines will plot as shown in Figure *7.3.

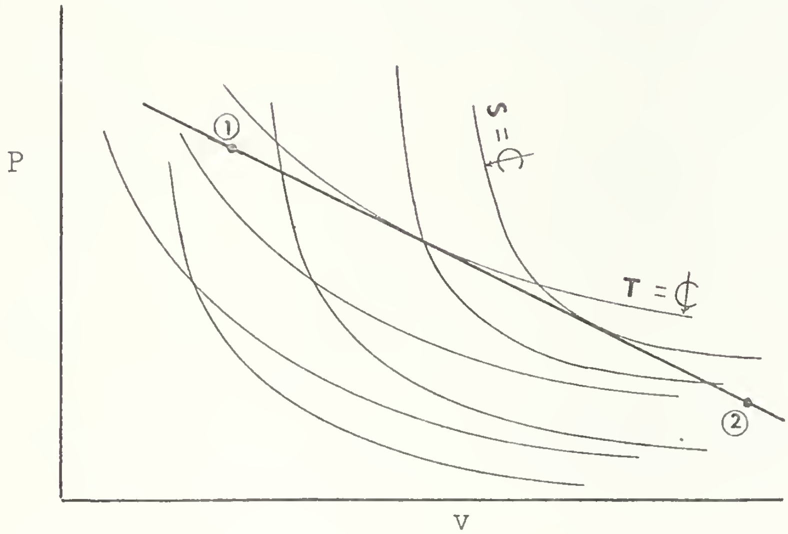



Figure *7.3

g. We now see that we can add more heat and go past the point of maximum temperature until we reach the point of maximum entropy. Let us pause for a moment and see if this appears reasonable.

Normally we think of the addition of heat as causing the fluid density to decrease. This requires the velocity to increase since $\rho V$ = constant. The increase in velocity requires a certain increase in kinetic energy. Thus, some of the heat which is added goes into increasing the

kinetic energy of the fluid and some increases the enthalpy (temperature) of the fluid. As we add more heat the required velocity increase (and corresponding kinetic energy increase) becomes larger and larger. Eventually we reach a point where all of the heat energy added is needed for the kinetic energy increase. At this point we reach a maximum enthalpy (temperature). Further addition of heat causes the kinetic energy to increase by an amount greater than the heat energy being added. Thus from this point on the enthalpy (temperature) decreases to provide the proper energy balance.

h. Can we go past the point of maximum entropy? Not by heat addition. An alternate way of stating this is that no more heat can be added to the system if we wish to maintain the same flow rate (i.e., stay on the same Rayleigh line). Thus, we see that a limit point has been reached and we say that the flow is "choked." We next proceed to investigate this limit point.

At the point of maximum entropy the slope of the 's = constant' line is the same as that of the Rayleigh line. Recall the equation of the Rayleigh line:

$$P + \frac{G^2 v}{g_c} = \text{Constant}$$

Differentiate to obtain

$$\frac{dP}{dv} = \frac{-G^2}{g_c} = \frac{-\rho^2 V^2}{g_c} \qquad (*7.2,12)$$

Recall the slope of the 's = constant' line:

$$\frac{dP}{dv} = -\gamma\frac{P}{v} = -\gamma\rho^2 RT \qquad (*7.2,11)$$

We now equate these:

$$-\frac{\rho^2 V^2}{g_c} = -\gamma\rho^2 RT \qquad (*7.2,13a)$$

and solve for

$$V^2 = g_c \gamma RT \qquad (*7.2,13b)$$

You immediately recognize this as sonic velocity and thus the 'limiting point' divides the Rayleigh line into a subsonic branch and a supersonic branch (similar to the situation in Fanno flow). Subsonic flow is on the left and supersonic flow is on the right of the limiting point in the P-v diagram.

j. Another interesting fact can be noted to exist at the limiting point. Recall the entropy relation from Section 1.10:

$$TdS = dh - \frac{dP}{\rho} \qquad (1.22)$$

The differential form of equation (10.2) may be written

$$dP = -\frac{\rho V dV}{g_c} \qquad (*7.2,14)$$

The property relation in equation (1.22) can thus be expressed as:

$$TdS = dh + \frac{VdV}{g_c} \qquad (*7.2,15)$$

At the 'thermal choke point' where M = 1, dS = 0, so:

$$0 = dh + \frac{VdV}{g_c} \qquad (*7.2,16)$$

Integrate this to obtain:

$$h + \frac{V^2}{2g_c} = \text{Constant} \qquad (*7.2,17)$$

We have already shown that this is the expression for 'stagnation enthalpy' (neglecting potential).

$$h_t = h + \frac{V^2}{2g_c} = \text{¢} \qquad \boxed{\text{at limit point ONLY!}} \qquad (*7.2,18)$$

Since at this 'limiting point', $h_t$ = constant, $dh_t$ must equal zero. Thus $h_t$ is a maximum at this point.

k. The above situations become more clear if the Rayleigh line is plotted in the h-s plane along with its associated stagnation curves. This has been done in Figure *7.4.

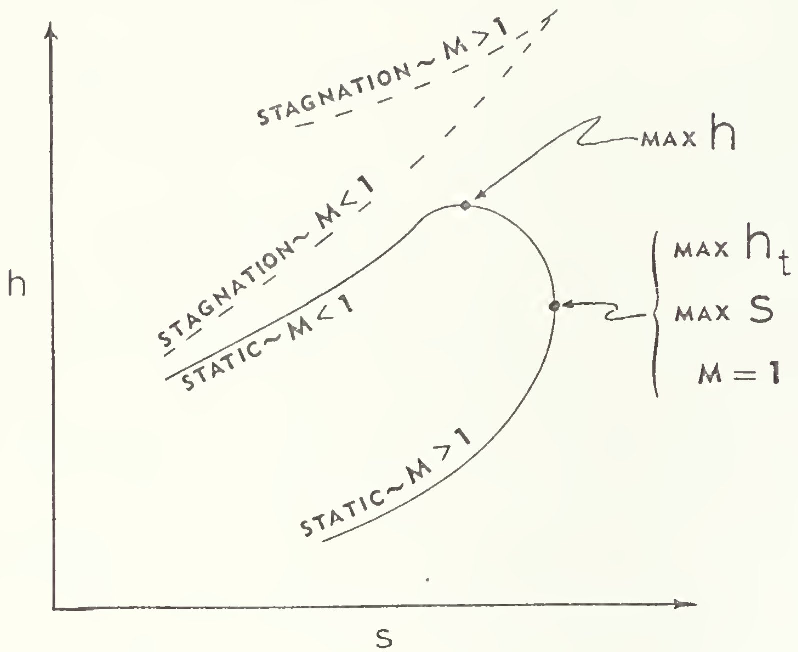



Figure *7.4

Rayleigh Line

Now correlate this plot with the previous information and satisfy yourself that the static and stagnation lines are properly drawn. Compare this figure to Figure *7.3. Can you add lines of constant pressure to Figure *7.4? Note that for a perfect gas this is equivalent to a T-s diagram. In which direction does a cooling process move along the subsonic branch? How do heating and cooling processes move on the supersonic branch?

## 7.3 - Working Relations for Raylegh Flow - Perfect Gas

a. We now proceed to develop property relations of a function of Mach number and specific heat ratio. These will be the working equations used for problem solving. We introduce the usual perfect gas assumptions as we have done many times previously.

b. From the momentum equation (*7.2,7) we can write:

$$P + \frac{\rho V^2}{g_c} = ¢ \qquad (*7.3,1)$$

but

$$V^2 = M^2 a^2 = M^2 \gamma g_c RT \qquad (*7.3,2)$$

and

$$\rho = \frac{P}{RT} \qquad (*7.3,3)$$

Show that

$$P(1 + \gamma M^2) = ¢ \qquad (*7.3,4)$$

Thus the pressure ratio between the two points is

$$\frac{P_2}{P_1} = \frac{1 + \gamma M_1{}^2}{1 + \gamma M_2{}^2} \qquad (*7.3,5)$$

How would you obtain a ratio for

$$\frac{P_{t_2}}{P_{t_1}} \qquad ??$$

Is this related to the entropy change as we have previously always been noting?

c. We saw that the continuity equation could be expressed as:

$$\rho V = \text{Constant}$$

Show that this can be written as:

$$\frac{PM}{\sqrt{T}} = \text{¢} \qquad (*7.3,6)$$

thus between two points

$$\frac{T_2}{T_1} = \left[ \frac{P_2}{P_1} \; \frac{M_2}{M_1} \right]^2 \qquad (*7.3,7)$$

Combining this with equation (*7.3,5) show that:

$$\frac{T_2}{T_1} = \left[ \frac{1 + \gamma M_1^{\,2}}{1 + \gamma M_2^{\,2}} \right]^2 \frac{M_2^{\,2}}{M_1^{\,2}} \qquad (*7.3,8)$$

d. In Unit 3 we saw that

$$T_t = T \left[1 + \frac{\gamma - 1}{2} M^2\right] \qquad (3.6) \text{ in text}$$

Now you can show that

$$\frac{T_{t_2}}{T_{t_1}} = \left[ \frac{1 + \gamma M_1^{\,2}}{1 + \gamma M_2^{\,2}} \right]^2 \frac{M_2^{\,2}}{M_1^{\,2}} \left[ \frac{1 + \frac{\gamma-1}{2} M_2^{\,2}}{1 + \frac{\gamma-1}{2} M_1^{\,2}} \right] \qquad (*7.3,9)$$

e. In summary, considering any two points in the flow system, if we know the properties at point 1 and <u>one</u> property at point 2 we can compute all other conditions at point 2.

f. Can we predict what will occur downstream?
Yes, if we have information concerning the heat transfer.

From the energy equation we know that

$$h_{t_1} + q_{1\to2} = h_{t_2} \qquad (*7.2,3)$$

Thus in general $q_{1\to2} = h_{t_2} - h_{t_1}$

and for perfect gases:

$$q_{1\to2} = c_p(T_{t_2} - T_{t_1}) \qquad (*7.3,10)$$

If we know the rate of heat transfer we can easily solve for $T_{t2}$ and then use equation (*7.3,9) to obtain $M_2$; and all the other properties are easily computed.

WARNING Note that: $q_{1\to2} = c_p \Delta T_t \neq c_p \Delta T$

g. The above developments assume $c_p$ = constant.
In some cases where heat transfer rates are extremely high with large temperature changes resulting, $c_p$ may vary enough to warrant using an average value for $c_p$. If, in addition, significant variations in $\gamma$ occur, it will be necessary to return to the basic equations and derive new working relations, treating $\gamma$ as a variable.

7.4 - Reference State and Tables

a. We again introduce a reference where the Mach number is unity — as reached by the process under consideration — i.e., by Rayleigh flow. We designate this the '*' condition. Note, in Figure *7.5, how this point is introduced in both the T-s diagram and the physical diagram, for a subsonic, heating example.

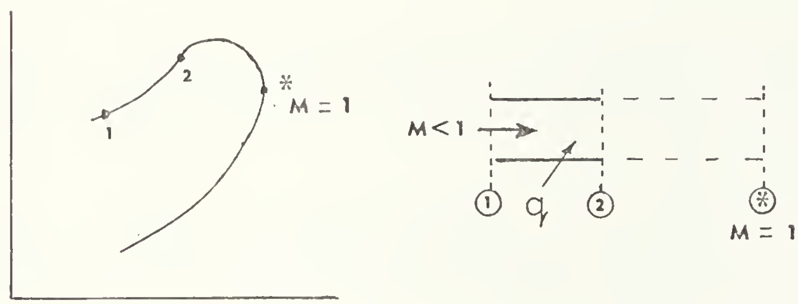



Figure *7.5

Complete the following diagrams (Figure *7.6) for a supersonic, cooling example.

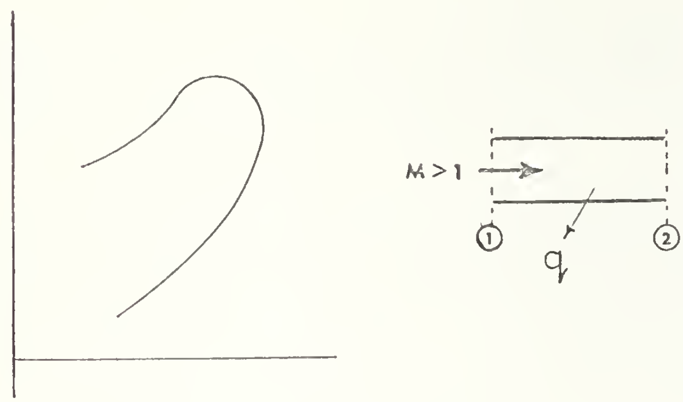



Figure *7.6

b. In the previous equations, let point (1) be the reference (or '*') condition and let (2) be any other point in the flow. Then ...

$M_1 = 1.0$ $\qquad$ $M_2 = M$

$P_1 = P^*$ , $T_1 = T^*$ $\qquad$ $P_2 = P$ , $T_2 = T$

$P_{t1} = P_t^*$, $T_{t1} = T_t^*$ $\qquad$ $P_{t_t} = P_t$, $T_{t_2} = T_t$

So

$$\frac{P_2}{P_1} = \frac{P}{P^*} = \frac{1 + \gamma}{1 + \gamma M^2} \qquad (*7.4,1)$$

Derive the expression for T/T*:

$$\frac{T_2}{T_1} = \frac{T}{T^*} \qquad ? \qquad (*7.4,2)$$

In like manner, expressions for the other ratios can be obtained and all of these are found tabulated in Appendix F in the text.

c. Pick a value for 'M' and let $\gamma$ be 1.4. Compute the corresponding value of T/T* and check your value in the tables.

d. Now read Section 10.2 and study the examples carefully. We will cover the remainder of Section 10 in the next unit which deals with the correlation of Rayleigh and Fanno flow.

e. Note that in Figure 10.5 in the text, the mass flow rate adjusts to a new Rayleigh line if heat is added to a duct in which the flow is "choked."

f. It will prove profitable to prepare a summary of the property changes that are taking place in the subsonic and supersonic regimes for the heating and cooling processes. Try to fill in the spaces in Table *7.1 without reference to the tables in Appendix F. Use basic equations, special equations and/or diagrams that have been developed for Rayleigh flow. For each property, indicate whether that property is increasing, decreasing or remaining constant for each regime.

TABLE *7.1

| Property | Heating | | Cooling | |
|---|---|---|---|---|
| | M < 1 | M > 1 | M < 1 | M > 1 |
| Stagnation Enthalpy (and temperature if a gas) | | | | |
| Entropy | | | | |
| Static Pressure | | | | |
| Velocity | | | | |
| Mach Number | | | | |
| Static Enthalpy (and temperature if a gas) | | | | |
| Stagnation Pressure | | | | |

Unit 8 - <u>CORRELATION OF FLOWS</u> - <u>OBJECTIVES</u>

The student shall be able to:

1. Compare similarities and differences among Fanno Flow, Rayleigh Flow and normal shocks.

2. Sketch on the same h-s diagram a Rayleigh Line, a Fanno Line and a normal shock process (all for the same flow per unit area).

3. Solve typical problems involving various sequential combinations of Fanno Flow, Rayleigh Flow and/or normal shocks.

## Unit 8 - CORRELATION OF FLOWS - STUDY GUIDE

### 8.1 Introduction.

a) In the previous developments, we have utilized the basic equations relating to fluid flow and have developed relations which have considered the effects of only one of these 'factors' at a time. We can gain a great deal more flexibility in problem solving by considering various sequential combinations of Fanno Flow, Rayleigh Flow and Normal Shocks.

b) Another type of problem involves simultaneous addition (or removal) of heat with friction effects of the same order of magnitude, and may even also include changes in area. Exact treatments for problems of this type usually involve tedious numerical integration as explicit solutions can not be obtained. Your text touches on some of these problems in sections 9.4, 9.5 and 10.5. You are encouraged to read these sections after you have completed this unit.

c) In this unit we wish to bring out some of the similarities and differences among Fanno Flow, Rayleigh Flow and Normal Shocks. Thus we shall assume steady, one-dimensional flow in a constant area duct with negligible changes in potential and no shaft work.

## 8.2 - Fanno Flow with a Normal Shock

a) Recall from Unit 7 the definition of the "Impulse" or "Thrust" Function:

$$PA + \frac{\dot{m}\ V}{g_c} = F \qquad (*7.2,7)$$

We saw that in a shock the two states (before and after the shock) displayed the same flow per unit area, the same stagnation enthalpy and the same impulse function. To corroborate this, re-examine equations *4.2,2; *4.3,3 and *4.2,6.

b) A Fanno Line represents states with the same flow per unit area ($\dot{m}/A$), and the same 'stagnation enthalpy' ($h_t$). As the flow passes along a Fanno Line, it is experiencing the effects of ____________? Because of this the 'Impulse Function' does not remain constant. Suppose we now plot the 'Impulse Function' versus Mach number for a Fanno Line. Such a plot is pictured in Figure *8.1.

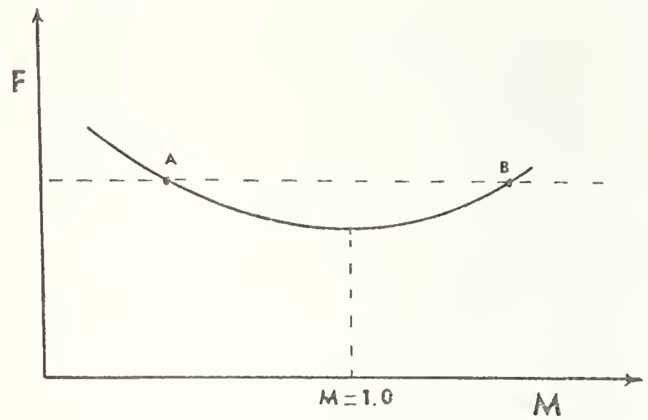



Figure *8.1

c) Note that for every point on the supersonic branch, there is a corresponding point on the subsonic branch where the Impulse Function has the same value. It should be obvious that points A and B satisfy all criteria for not only the Fanno Line but also for the Normal Shock.

d) Thus we can imagine a shock between points A and B in Figure *8.1. Would this shock process occur from A to B or from B to A ?

e) Consider a constant area duct such as in Figure *8.2a which is experiencing Fanno Flow. We can now visualize the possibility of a shock in the duct as pictured between points (2) and (3). This situation meets the conditions of flow along the supersonic branch of a given Fanno Line from point (1) to point (2) and flow along the subsonic branch of the <u>same</u> Fanno Line from (3) to (4). We can also plot this situation in the h-s plane as shown in Figure *8.2b.

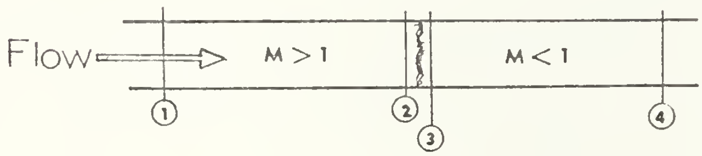



Figure *8.2a

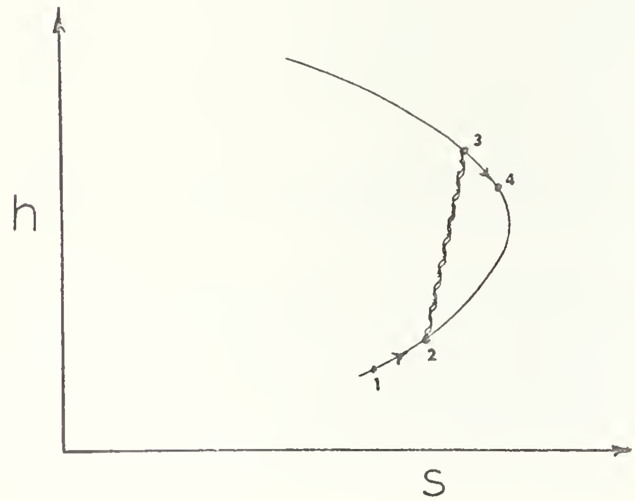



Figure *8.2b

8.3 - Rayleigh Flow with a Normal Shock

a) A Rayleigh Line represents states which exhibit the same flow per unit area and the same impulse function but do not have a constant stagnation enthalpy, due to heat transfer. We can picture a Rayleigh Line and its corresponding Stagnation Enthalpy line in the h-s plane as shown in Figure *8.3.

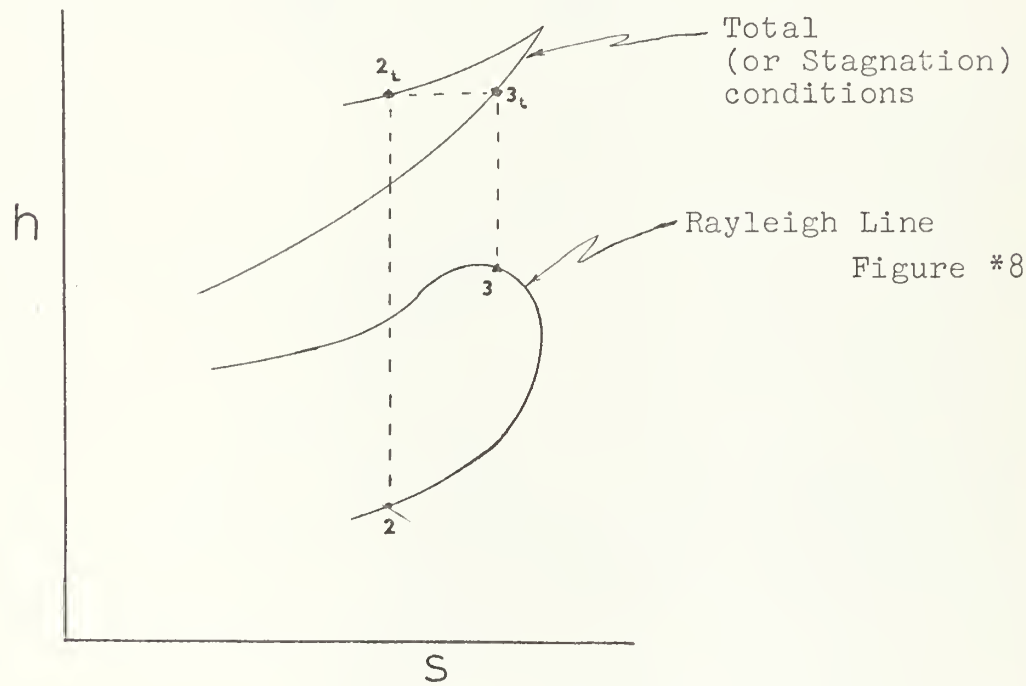



Figure *8.3

Notice that for every point on the supersonic branch of the Rayleigh Line there is a corresponding point on the subsonic branch which has the same stagnation enthalpy. (Also note that the converse is not always true). Would $q_{2\to *}$ be greater than, less than or equal to $q_{3\to *}$? Why (or why not)?

b) As with the Fanno Flow and a Normal Shock, we can now surmise the existence of a duct experiencing Rayleigh Flow and a Normal Shock forming as pictured in Figure *8.4a. The corresponding process is shown in the h-s plane in Figure *8.4b. Note that the flow from point 1 to point 2 and the flow from points 3 to 4 lie on the same Rayleigh Line.

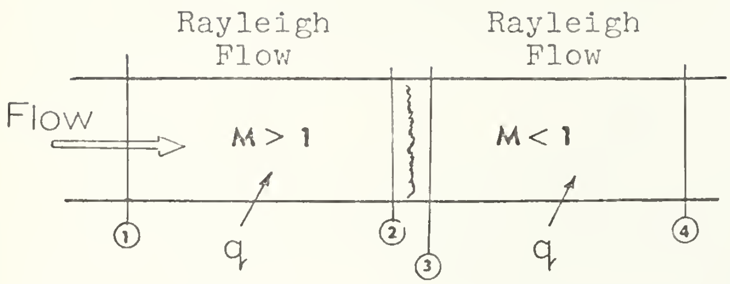



Figure *8.4a

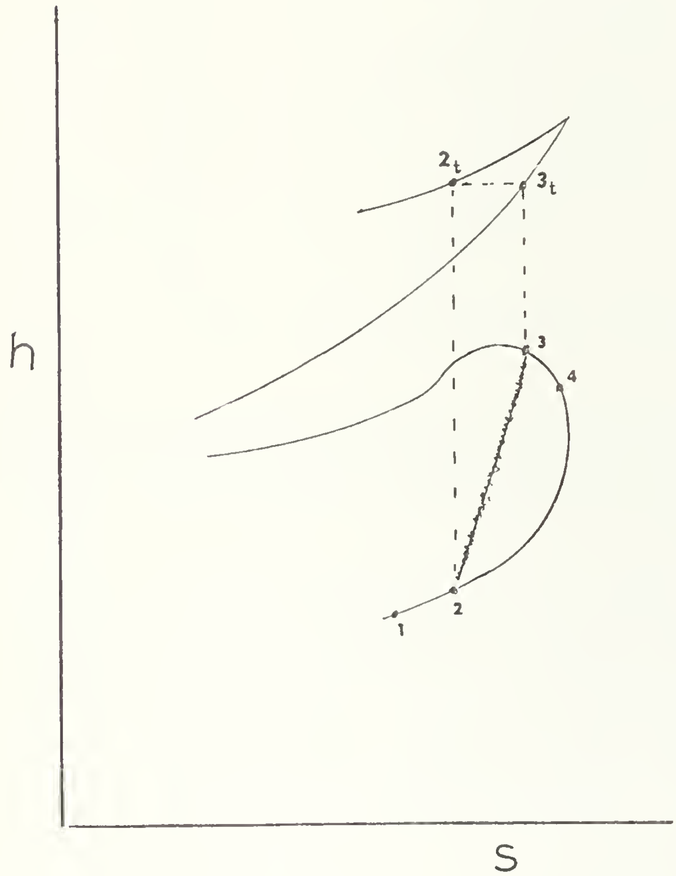



Figure (8.4b

## 8.4 Rayleigh Flow, Fanno Flow and Normal Shocks

a) Consider now the combination of Rayleigh Flow, Fanno Flow and a shock process all for the same mass flow per unit area, as pictured in the h-s plane in figure *8.5.

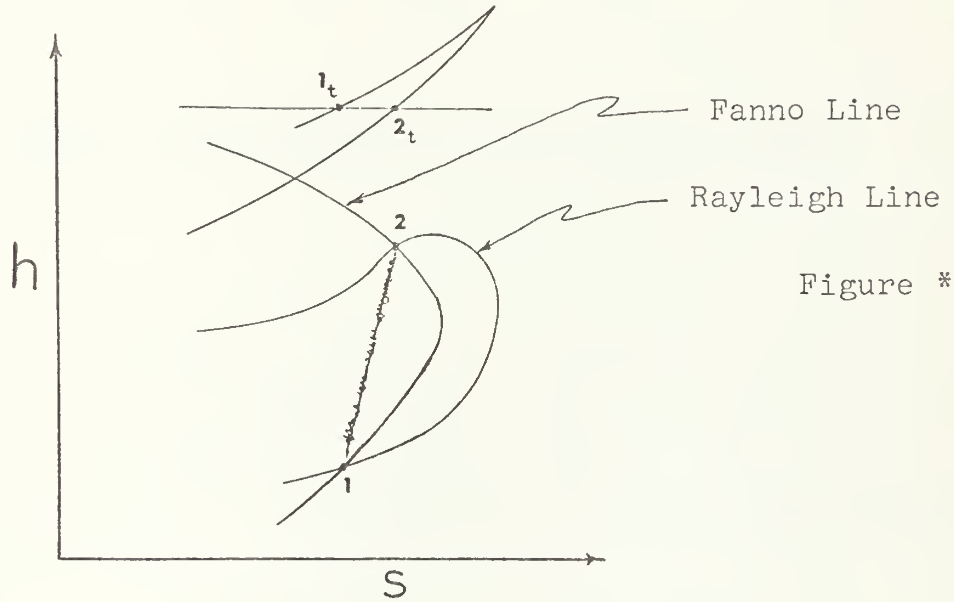



Figure *8.5

b) We have seen that points (1) and (2) have the same mass flow rate, the same impulse function, and the same stagnation enthalpy; and thus can lie on a Fanno Line, a Rayleigh Line, and at opposite sides of a Normal Shock.

c) From the units on Rayleigh and Fanno flows we saw that when the "choked flow" condition was reached along the subsonic flow branch, the mass flow rate re-adjusted to provide for flow along a <u>new</u> Rayleigh Line (or Fanno Line) if more heat (or duct length, hence friction) was added to the flow. In the supersonic regime the readjustment can (and usually does) take the form of a normal shock. In this case we do not necessarily have to move to a <u>new</u> Fanno or Rayleigh Line since once subsonic flow has been obtained we can add much more heat, or introduce much more friction, as the case may be. Many problems can be considered by matching these three types of flow processes sequentially, knowing the flow conditions. Let us now consider <u>qualitatively</u> two such possible combinations.

d) Consider the combination in figure *8.6a. Assume that only when heat is being transferred may friction effects be considered negligible.

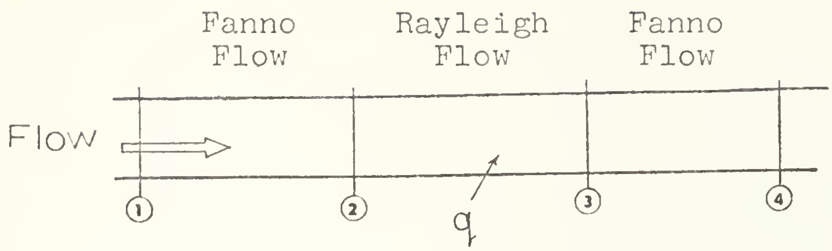



Figure *8.6a

If this entire flow system is in the subsonic regime the h-s diagram would be that shown in Figure *8.6b.

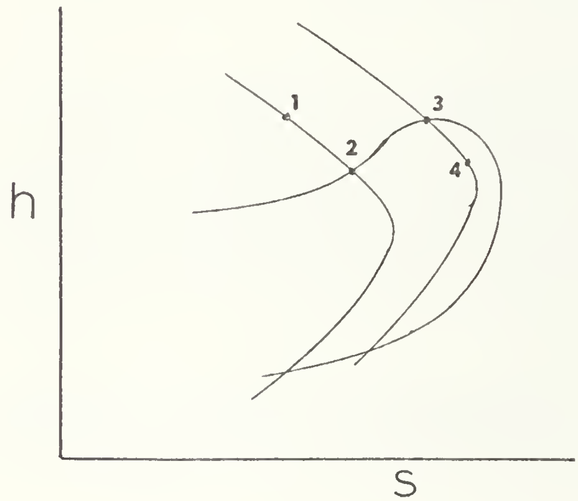



Figure *8.6b

We may describe the process as follows:

1) From (1) to (2) the flow is along a Fanno Line hence '$h_t$' and '$\dot{m}/A$' are constant.

2) From (2) to (3) '$\dot{m}/A$' remains unchanged as does 'F' (i.e., '$PA + \dot{m}V/g_c$'), but due to the heat transfer, '$h_t$' is increasing.

3) From (3) to (4) the flow is along a different Fanno Line even though it has the same '$\dot{m}/A$'. The reason is that the stagnation enthalpies for the two Fanno Lines are different.

e. Now suppose we consider the situation in Figure *8.6a but with supersonic flow coming in at point (1). Plot this situation in the h-s plane in Figure *8.6c assuming that the flow is not choked.


Figure
*8.6c

f) We can easily imagine the amount of heat added from (2) to (3) exceeding that required to "choke" the flow, in which case a shock could possibly form. Many interesting possibilities could result depending upon the boundary conditions imposed on the system. Are the original stagnation conditions maintained? If the flow is subsonic at the exit, what pressure must exist?

g) It might prove useful to summarize the effect on fluid properties of flow in ducts in the supersonic and subsonic regimes for the various types of flow.

Table *8.1

| Property | Rayleigh Flow | | | | Fanno Flow | | Normal Shock |
|---|---|---|---|---|---|---|---|
| | Heating | | Cooling | | | | |
| | $M<1$ | $M>1$ | $M<1$ | $M>1$ | $M<1$ | $M>1$ | $M>1$ |
| Pressure | D | I | I | D | D | I | I |
| Mach | I | D | D | I | I | D | to $M<1$ |
| Velocity | I | D | D | I | I | D | D |
| Temperature | I/D | I | I/D | D | D | I | I |
| Entropy | I | I | D | D | I | I | I |
| Stag. Press. | D | D | I | I | D | D | D |
| Stag. Temp. | I | I | D | D | | | |
| Density | D | I | I | D | D | I | I |
| Impulse Fct. | | | | | D | D | |

Check this table with the information developed in the previous units and satisfy yourself that the information is correct.

h) Now read Section 10.3 in the text. If time permits, glance at Sections 9.4, 9.5 and 10.5.

APPENDIX B -

STUDENT EVALUATION QUESTIONAIRE †

AE 3043, "Gas Dynamics" - Quarter IV, 1971-72

Please answer these questions as completely and candidly as possible. The results will in NO fashion effect your grade, but will be utilized in assessing the value and desirability of this method of instruction.

Where you are given a scale on which to indicate your opinion, a descriptive work (or words) is provided to define each extreme. Please consider the scale linear with the mean midway between 2 and 3. You may thus circle 1, 2, 3 or 4 to indicate your own attitude relative to the extremes. Any amplifying comments you may wish to add to any question will be greatly appreciated. (Use the back of the questionaire or attach additional pages, as desired)

1. How many previous courses (of any duration, here at NPS or elsewhere) have you taken which were...

   SELF STUDY $\boxed{\text{Ave. 1.9}}$ ? SELF PACED $\boxed{\text{Ave. 2.9}}$ ?

   PROGRAMMED $\boxed{\text{75\% had had many}}$ ?

2. Indicate your opinion of the utility of the following educational innovations (NOTE: These features may or may NOT have been included in this course.)

| | | | | | | | | Average Rating |
|---|---|---|---|---|---|---|---|---|
| a. Defined Objectives | (no value) | 1 | 2 | 3 | 4 | (great value) | | 3.50 |
| b. Self-Study Concept | (no value) | 1 | 2 | 3 | 4 | (great value) | | 3.08 |
| c. Self-Paced Instruction | (no value) | 1 | 2 | 3 | 4 | (great value) | | 3.00 |
| d. Study Guide | (no value) | 1 | 2 | 3 | 4 | (great value) | | 3.67 |
| e. Check Tests | (no value) | 1 | 2 | 3 | 4 | (great value) | | 3.08 |
| f. Group Discussions | (no value) | 1 | 2 | 3 | 4 | (great value) | | 3.18 |
| g. Individual Tutoring | (no value) | 1 | 2 | 3 | 4 | (great value) | | 2.92 |
| h. Audio-Tutorial (Tape-Slides) | (no value) | 1 | 2 | 3 | 4 | (great value) | | 2.75 |
| i. Final Exam | (no value) | 1 | 2 | 3 | 4 | (great value) | | 2.67 |

†Boxed values indicate the average results of Questionnaires returned to the writer.

STUDENT EVALUATION QUESTIONAIRE (page 2.)

The following questions pertain to the materials/methods used in this course. Please consider each question as it reflects on the course as a whole, then circle or underline your opinion. Then below each question, identify any unit (or section) to which your general answer does not apply and explain why. You will probably find it useful to have your course materials at hand while considering these questions.

3. How would you rate the clarity of the Course OBJECTIVES?

(obscure) 1 2 3 4 (clear)
Exceptions:

Average rating - 3.58

4. Did you find the Study Guides understandable?

(obscure) 1 2 3 4 (clear)
Exceptions:

Average rating - 3.51

5. What is your opinion of the organization of the Study Guides?

(confused) 1 2 3 4 (logical)
Exceptions:

Average rating - 3.42

6. Were sufficient graphs/diagrams/etc. provided in the Study Guides?

(far too few) 1 2 3 4 (far too many)
Exceptions:

Average rating - 2.42

7. Were there enough example problems provided in the Study Guides?

(far too few) 1 2 3 4 (far too many)
Examples:

Average rating - 2.17

STUDENT EVALUATION QUESTIONAIRE (page 3.)

8. Were there enough homework problems provided to cover the material?

(far too few) 1 2 3 4 (far too many)
Exceptions:

Average rating - 3.04

9. Was a correct portion of the quarter allocated for the completion of each unit in the course:

(too little time) 1 2 3 4 (too much time)
Exceptions:

Average rating - 2.58

10. Outside of the scheduled problem sessions, on the average, how many times did you consult the instructor for advice/assistance, per unit? ____________

Exceptions:

Average - 1 time

11. Did you seek advice/assistance from your classmates?

(seldom) 1 2 3 4 (regularly)
Exceptions:

Average rating - 2.5

12. How do you feel the course materials aided you in preparing for the check tests?

(useless) 1 2 3 4 (invaluable)
Exceptions:

Average rating - 3.58

13. How beneficial were the check tests in preparing for the major quizzes/final exam?

(useless) 1 2 3 4 (invaluable)
Exceptions:

Average rating - 3.08

14. Estimate the percentage of your total study time expended on this course. __________

| Average rating - 25% |
|---|

15. Do you feel any unit was excessively long or short? (If so please identify and comment.)

| | |
|---|---|
| Unit one | - 2 |
| Unit four | - 2 |

16. How do you feel the amount you learned using this controlled, self-study method compares with what you would have learned in a conventional lecture course? (less in this course; about the same; more in this course)

| less in this course | about the same | more in this course |
|---|---|---|
| 1 | 7 | 4 |

17. For several reasons this course was run on a controlled-pace basis.

a. Would you have liked the course better if it were completely self-paced? ________

| | |
|---|---|
| Yes | - 5 |
| No | - 6 |

b. If it were run on a completely self-paced basis, when do you think you would have finished the course?

| | |
|---|---|
| on time | - 7 |
| early | - 2 |
| late | - 2 |

18. If you took a self-paced course and you were among the first students to successfully finish a unit, would you be willing to serve as a tutor to those having more difficulty? (please comment)

| | |
|---|---|
| Yes | - 11 |
| No | - 1 |

19. Assuming check tests would be retaken until mastery of material is essentially demonstrated, and that failure of a check test would not be held against you - would it be better to have more comprehensive check tests and eliminate the major quizzes? (please comment)

| | |
|---|---|
| Yes | - 6 |
| No | - 6 |

20. Would you prefer check tests to be ...

| | |
|---|---|
| closed book? | 7 |
| open book? | 0 |
| part open, part closed | 4 |
| oral? | 1 |

21. To what extent has participation in this course improved your ability to "decipher" a typical textbook or technical paper?

| | |
|---|---|
| little | 8 |
| much | 2 |

22. Given the choice of some type of self-study method as opposed to instruction using the conventional lecture method, which would you prefer?
(You may qualify your answer in any way you choose... but please give your reasons. This is your chance to comment on anything you feel this questionnaire overlooked.)

| | |
|---|---|
| self study | 9 |
| lecture | 2 |

23. Would you like to take all your courses by some type of a self-study method?

| | |
|---|---|
| no | - 6 |
| yes | - 3 |
| undecided | - 3 |

THANK YOU FOR YOUR TIME AND EFFORT!

APPENDIX C -

SELECTED COMMENTS EXTRACTED FROM STUDENT QUESTIONNAIRES AND CRITIQUE SESSION.

Question 19

"... yes... eliminate the pressure of having to do well at a specified time..."

"Yes. Retakes under present check test, hour quiz policy are senseless waste of time."

"No because it is very tiresome getting up for a test every week"

"No. The check tests are aids, but the quizzes are for grades. (forgive my poetic license)"

Question 21

> Comment by author:
> In general the students seemed to miss the intent of this question. Several commented that they had no basis on which to answer. Most answers seemed hesitant, or non-commital.

Question 22

"Self study involving lesson guides, (audio-visual perhaps)
Prob sets
Followed by active participation seminar to reinforce important points"

"Prefer a lecture. I have a good memory so I can keep up with previous ideas while learning others. Most lecturers speak at an easily grasped pace for me."

> Comment by author:
> 9 of the 12 questionnaires indicated a preference for this method or some modification of self-paced instruction.

Comment from Critique Session

"... This was really not what I would call self-taught but was more like an ideal lecture course, where everyone has read his lesson assignment before class and the material is re-inforced by the professor in the lecture..."

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13. ABSTRACT

A personalized system of instruction combining the principle of auto-tutorial instruction with modified self-pacing was applied to a course in the fundamentals of gas dynamics taught to thirteen students in the Department of Aeronautics at the Naval Postgraduate School, Monterey, during the Spring quarter of 1972. The course results are summarized and the instructional materials developed for the course are included as appendices. While some degradation was apparent due to modifications made to the method, the results tended to affirm the substantial value of this instructional method and its superiority to the conventional lecture method.

| 14 KEY WORDS | LINK A | | LINK B | | LINK C | |
|---|---|---|---|---|---|---|
| | ROLE | WT | ROLE | WT | ROLE | WT |
| Self-Paced | | | | | | |
| Proctorial | | | | | | |
| Personalized Instruction | | | | | | |
| Tutorial Instruction | | | | | | |
| Rayleigh | | | | | | |
| Fanno Flow | | | | | | |
| Gas Dynamics | | | | | | |

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